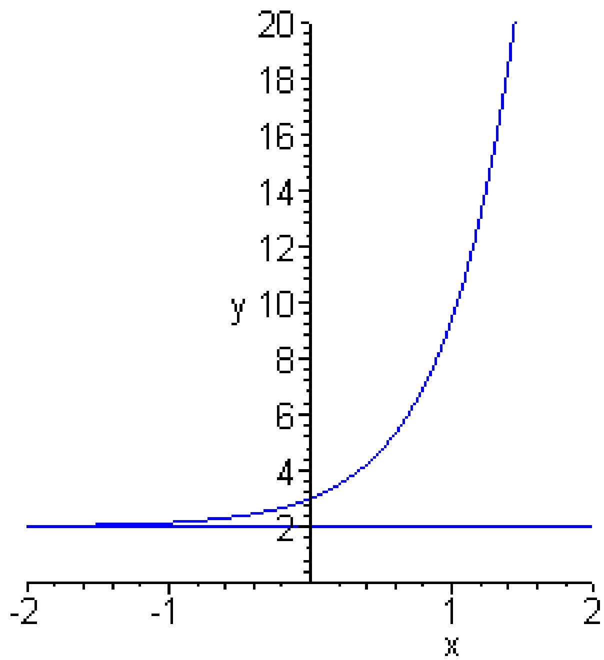
1. a. If $e^a = 3.7$ and $e^b = 0.4$, then

$$\begin{aligned}
\frac{(e^0 + e^a)^2}{e^{a-b}} &= \frac{e^b(1+e^a)^2}{e^a} \\
&= \frac{0.4(1+3.7)^2}{3.7} \\
&= \frac{0.4(4.7)^2}{3.7} = 2.3881.
\end{aligned}$$

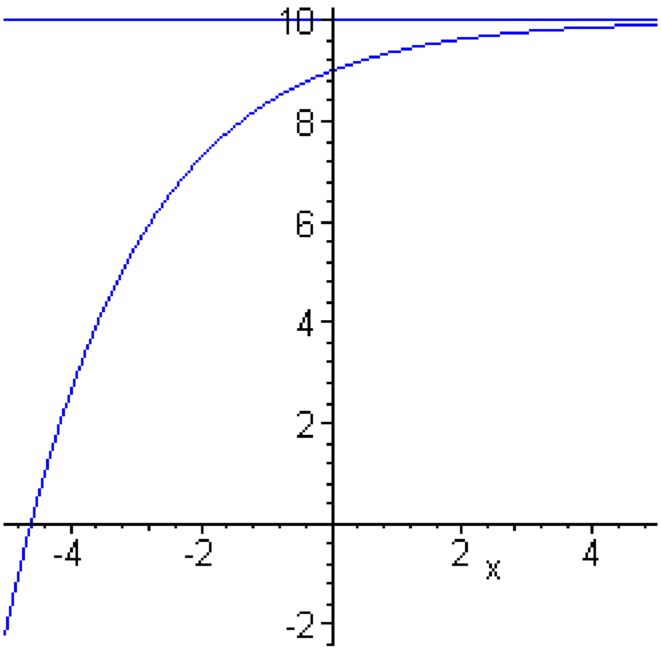
b. If $\ln(c) = -1.5$ and $\ln(d) = 2$, then

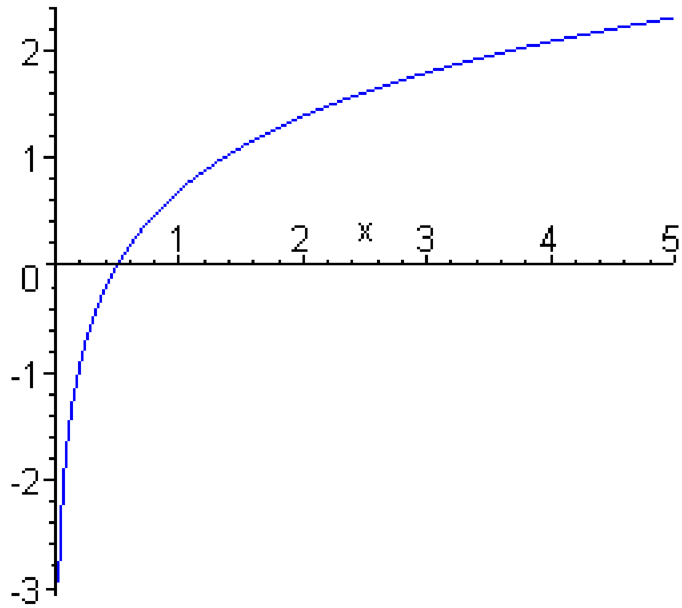
$$\begin{aligned}
\frac{\ln(d^2/c) - \ln(e)}{(\ln(cd) + \ln(1))} &= \frac{\ln(d^2) - \ln(c) - 1}{\ln(c) + \ln(d) + 0} \\
&= \frac{2\ln(d) - ln(c) - 1}{\ln(c) + ln(d)} \\
&= \frac{2(2.1) - (-1.5) - 1}{-1.5 + 2.1} = \frac{4.7}{0.6} = 7.833.
\end{aligned}$$

4. The x-intercept would satisfy $f(x) = 2 + e^{2x} = 0$, but both 2 and e^{2x} are positive, $f(x) = 0$ is impossible and no x-intercept exists. The y-intercept is where $x = 0$, so $f(0) = 2+e^0 = 3$. Thus, it occurs at $(0, 3)$. The exponential is defined for all x, so there are no vertical asymptotes. A horizontal asymptote is found by examining $x \to -\infty$. As $x \to -\infty$, $e^{2x} \to 0$, so $y \to 2+0 = 2$. It follows that there is a horizontal asymptote (to the left) at $y = 2$. The graph is shown below to the left.



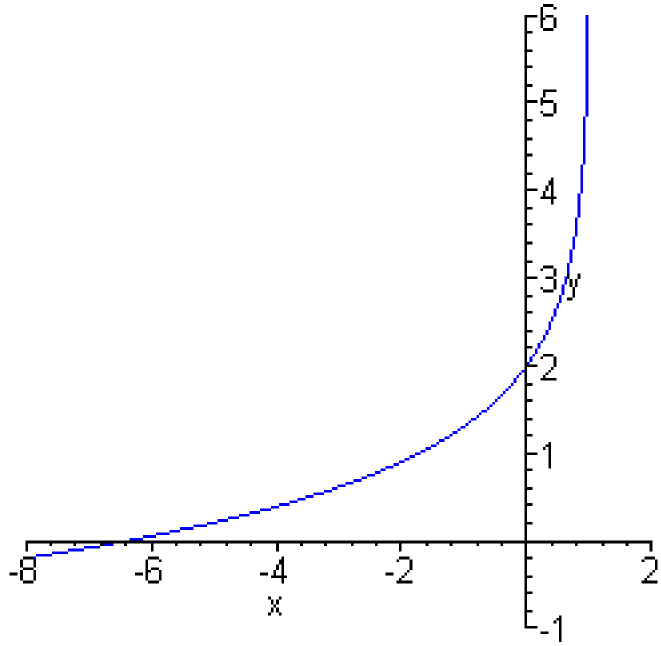
5. The x-intercept satisfies $f(x) = 10 - e^{-x/2} = 0$, so $e^{-x/2} = 10$. By taking the logarithms of both sides, it follows that $\frac{-x}{2} = \ln(10)$, so $x = -2\ln(10) \simeq -4.605$ and the x-intercept is $(-2\ln(10), 0)$. The y-intercept is where $x = 0$, so $f(0) = 10 - e^0 = 9$. Thus, it occurs at $(0, 9)$. The exponential is defined for all x, so there are no vertical asymptotes. A horizontal asymptote is found by examining $x \to +\infty$. As $x \to +\infty$, $e^{-x/2} \to 0$, so $y \to 10 - 0 = 10$. It follows that there is a horizontal asymptote (to the right) at $y = 10$. The graph is shown below to the left.





7. The domain requires that the argument of the logarithm is positive, so $2x > 0$ or $x > 0$. The x-intercept satisfies $f(x) = \ln(2x) = 0$, so $\ln(2x) = 0$ or $2x = e^0 = 1$ Thus, $x = \frac{1}{2}$, and the x-intercept occurs at $(\frac{1}{2}, 0)$ The y-intercept is where $x = 0$, but this is outside the domain, hence doesn't exist. A vertical asymptote occurs on the edge of the domain or at $x = 0$. The graph is shown above to the right.

10. The domain requires that the argument of the logarithm is positive, so $1-x>0$ or $x<1$. The x-intercept satisfies $f(x)=2-\ln(1-x)=0$, so $\ln(1-x)=2$ or $1-x=e^2$ Thus, $x=1-e^2$, and the x-intercept occurs at $(1-e^2,0)$ The y-intercept is where $x=0$, so $f(0)=2-\ln(1-0)=2$ or $(0,1)$. A vertical asymptote occurs on the edge of the domain or at $x=1$. The graph is shown below.

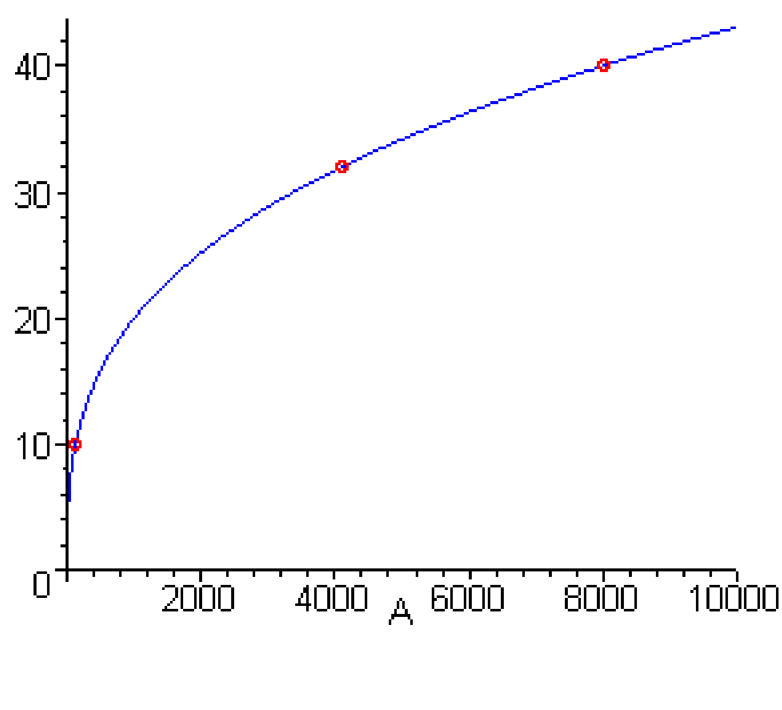


11. a. The average number of mammalian species satisfies $N=kA^{\frac{1}{3}}$, so if the islands have areas of 125 and 8000 km^2, then

$$\begin{aligned} N(125) &= 2(125)^{\frac{1}{3}}=2(5)=10, \\ N(8000) &= 2(8000)^{\frac{1}{3}}=2(20)=40. \end{aligned}$$

b. For 32 species of mammals, the island area satisfies $N=32=2A^{\frac{1}{3}}$, so $16=A^{\frac{1}{3}}$ and $A=16^3=4096\text{km}^2$.

c. The graph is below.



14. a. The allometric model satisfies:

$$\begin{aligned} P &= kW^a, \\ \ln(P) &= a\ln(W) + \ln(k). \end{aligned}$$

Note that if $Y = \ln(P)$, $X = \ln(W)$, and $K = \ln(k)$, then this is just the equation of a line

$$Y = aX + K,$$

where X and Y are the logarithms of the data.
The data given is shown with their logarithmic values in the table below:

W	$\ln(W)$	P	$\ln(P)$
4	1.386	615	6.422
28	3.332	350	5.858

From the formula above, the slope is a and satisfies

$$a = \frac{\ln(P_2) - \ln(P_1)}{\ln(W_2) - \ln(W_1)} = \frac{5.858 - 6.422}{3.332 - 1.386} = -0.2897.$$

To obtain k, we see that

$$\ln(k) = \ln(P_1) - a\ln(W_1) = 6.422 + 0.2897(1.386) = 6.823,$$

so

$$k = e^{\ln(k)} = e^{6.823} = 918.9.$$

This gives the allometric model

$$P = 918.9W^{-0.2897}.$$

b. For an 11 gram wren, the allometric model gives:

$$P = 918.9(11)^{-0.2897} = 459 \text{ beats/min}.$$

If a dove has a pulse of 130 beats/min, then the allometric model gives

$$\begin{aligned} 130 &= 918.9W^{-0.2897} \\ W^{0.2897} &= \frac{918.9}{130} \\ W &= \left(\frac{918.9}{130}\right)^{1/0.2897} = 855 \text{ g} \end{aligned}$$

15. a. The allometric model satisfies:

$$\begin{aligned} T &= kw^a, \\ \ln(T) &= a\ln(w) + \ln(k). \end{aligned}$$

Note that if $Y = \ln(T)$, $X = \ln(w)$, and $K = \ln(k)$, then this is just the equation of a line

$$Y = aX + K,$$

where X and Y are the logarithms of the data.
The data given is shown with their logarithmic values in the table below:

w	$\ln(w)$	T	$\ln(T)$
70	4.248	120	4.787
1.5	0.4055	65	4.174

From the formula above, the slope is a and satisfies

$$a = \frac{\ln(T_2) - \ln(T_1)}{\ln(w_2) - \ln(w_1)} = \frac{4.787 - 4.174}{4.248 - 0.4055} = 0.1595.$$

To obtain k, we see that

$$\ln(k) = \ln(T_1) - a\ln(w_1) = 4.787 - 0.1595(4.248) = 4.110,$$

so

$$k = e^{\ln(k)} = e^{4.110} = 60.93.$$

This gives the allometric model

$$T = 60.93w^{0.1595}.$$

b. For a 20 kg dog, the allometric model gives:

$$T = 60.93(20)^{0.1595} = 98.3 \text{ days}.$$

If an animal has erythrocytes with a lifetime of 100 days, then the allometric model gives

$$\begin{aligned} 100 &= 60.93w^{0.1595} \\ w^{0.1595} &= \frac{100}{60.93} \\ w &= \left(\frac{100}{60.93}\right)^{1/0.1595} = 22.3 \text{ kg} \end{aligned}$$