Math 121 **Algebra Self-Test - Complete Solutions** **Spring 2001**

If you feel that my solutions are omitting too many steps, then you are probably unprepared for Math 121. You should know all steps for solving these problems.

1. From the equation $4x - 3 = 7 - x$, we see that $4x + x = 7 + 3$ or $5x = 10$. Thus, $x = 2$.

2. From the equation $x^2 + 7 = 1 - 5x$, we have $x^2 + 5x + 6 = 0$. This can be factored to give $(x + 2)(x + 3) = 0$, which gives $x = -2, -3$.

3. From the equation $2 - 0.3x = 2.1x + 5.6$, we see that $2.1x + 0.3x = 2 - 5.6$ or $2.4x = -3.6$. Thus, $x = -1.5$.

4. From the equation $3x^2 - 10 = 8 + 3x$, we have $3x^2 - 3x - 18 = 3(x^2 - x - 6) = 0$. This is easily factored to $3(x - 3)(x + 2) = 0$, which gives $x = 3, -2$.

5. From the equation $\frac{2}{x+4} = \frac{4}{2-x}$, we can cross multiply to give $2(2 - x) = 4(x + 4)$, so $4 - 2x = 4x + 16$ or $6x = -12$. Thus, the solution is $x = -2$.

6. From the equation $\frac{3}{x-4} = 2 - \frac{1}{x+2}$, we first find a common denominator for the right hand side giving $\frac{3}{x-4} = \frac{2(x+2)-1}{x+2} = \frac{2x+3}{x+2}$. Next we cross multiply, giving $3(x + 2) = (2x + 3)(x - 4)$ or $3x + 6 = 2x^2 - 5x - 12$. Thus, $2x^2 - 8x - 18 = 2(x^2 - 4x - 9) = 0$. We apply the quadratic formula to $x^2 - 4x - 9 = 0$, giving $x = \frac{4 \pm \sqrt{16+36}}{2} = 2 \pm \sqrt{13}$.

7. From the equation $\frac{x}{3} + 4 = \frac{2x}{5} - \frac{7}{2}$, we see that $(\frac{2}{5} - \frac{1}{3})x = 4 + \frac{7}{2}$ or $\frac{1}{15}x = \frac{15}{2}$. Thus, $x = \frac{225}{2} = 112.5$.

8. From the equation $\frac{2}{x(x+2)} = \frac{1}{2x+3}$, we cross multiply to give $2(2x + 3) = x(x + 2)$ or $4x + 6 = x^2 + 2x$. This gives the quadratic equation $x^2 - 2x - 6 = 0$. The quadratic formula gives $x = \frac{2 \pm \sqrt{4+24}}{2} = 1 \pm \sqrt{7}$.

9. The equation $10^{x-3} = 100$ can be written $10^{x-3} = 10^2$. We equate the exponents, giving $x - 3 = 2$, so $x = 5$.

10. The equation $\frac{1}{2^{x+2}} = 8$ can be written $2^{-(x+2)} = 2^3$. Again we equate the exponents, giving $-x - 2 = 3$. Thus, $x = -5$.

11. The equation $\log_{10}(x + 20) = 3$ can be written $x + 20 = 10^3 = 1000$. Thus, $x = 980$.

12. The equation $\log_2 32 = x - 1$ can be written $32 = 2^{x-1}$ or $2^{x-1} = 2^5$. It follows that $x - 1 = 5$, so $x = 6$.

13. The equation $2^7 + 2^7 = 2^x$ can be written $2^x = 2^7(1 + 1) = 2^8$. Thus, $x = 8$.

14. From properties of logarithms, the equation $\log(x + 4) - \log(2) = \log(x)$ can be written $\log\left(\frac{x+4}{2}\right) = \log(x)$. It follows that $\frac{x+4}{2} = x$ or $x + 4 = 2x$. Thus, $x = 4$.

15. From the equation $|2x+4| = 16$, either $2x+4 = 16$,, in which case $x = \frac{(16-4)}{2} = 6$, or $-(2x+4) = 16$, in which case $x = \frac{(-16-4)}{2} = -10$

16. For the equation $\left|\frac{x}{2} - 3\right| = 2x + \frac{1}{3}$, we first note that $2x + \frac{1}{3} \geq 0$ or $x \geq -\frac{1}{6}$. Then either $\frac{x}{2} - 3 = 2x + \frac{1}{3}$ or $\frac{-x}{2} + 3 = 2x + \frac{1}{3}$. In the first case, $2x - \frac{x}{2} = -3 - \frac{1}{3}$ or $\frac{3x}{2} = -\frac{10}{3}$, so $x = -\frac{20}{9}$, but this is not a solution, since $x < -\frac{1}{6}$. The second case gives $2x + \frac{x}{2} = 3 - \frac{1}{3}$ or $\frac{5x}{2} = \frac{8}{3}$, so $x = \frac{16}{15}$.

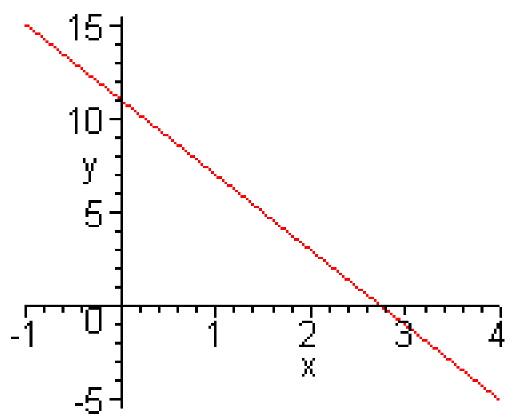
17. The equation $3x - 4 < 5$ implies $3x < 9$, so $x < 3$.

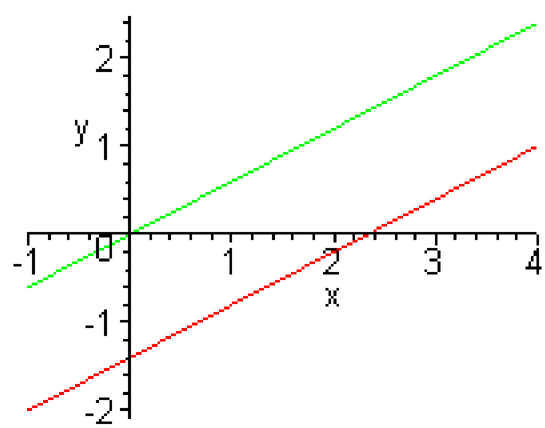
18. The equation $x^2 - 2x - 3 \geq 0$ satisfies $(x-3)(x+1) \geq 0$. So either both factors are positive, which implies $x \geq 3$, or both factors are negative, which implies $x \leq -1$.

19. The equation $x^2 + 2x < 5$ is equivalent to $x^2 + 2x - 5 < 0$. By applying the quadratic formula to $x^2 + 2x - 5 = 0$, we obtain $x = \frac{-2 \pm \sqrt{4+20}}{2} = -1 \pm \sqrt{6}$. For the inequality to hold, one factor must be positive and the other negative, so $-1 - \sqrt{6} < x < -1 + \sqrt{6}$.

20. To analyze the equation $\frac{2}{x} < -\frac{4}{x+6}$, we want to cross multiply. However, we must know if the quantities in the denominator are positive or negative. If $x > 0$, then both denominators are positive, so we have $2(x+6) < -4x$ or $6x < -12$ or $x < -2$. Thus, there are no solutions with $x > 0$. If $-6 < x < 0$, then the denominator on the left is negative and the denominator on the right is positive. Thus, $2(x+6) > -4x$ or $x > -2$. Finally, if $x < -6$, then both denominators are negative. Thus, $2(x+6) < -4x$ or $x < -2$. This implies that $x < -6$ satisfies the inequality. In summary, the solution is all x such that either $x < -6$ or $-2 < x < 0$.

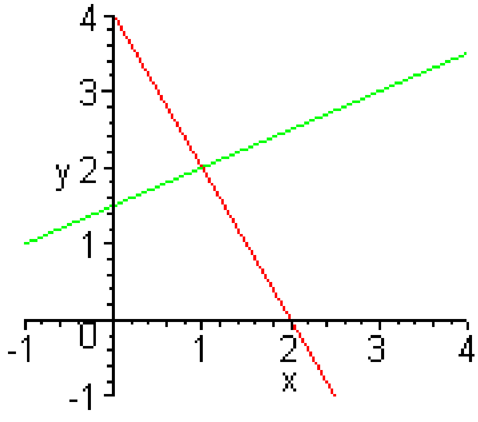
21. The slope of the line is $m = \frac{-1-7}{3-1} = -4$. Use $y = mx + b$ with one point $7 = -4(1) + b$ to find $b = 11$. Thus, the equation of the line is $y = -4x + 11$. Below left is a graph of the line.

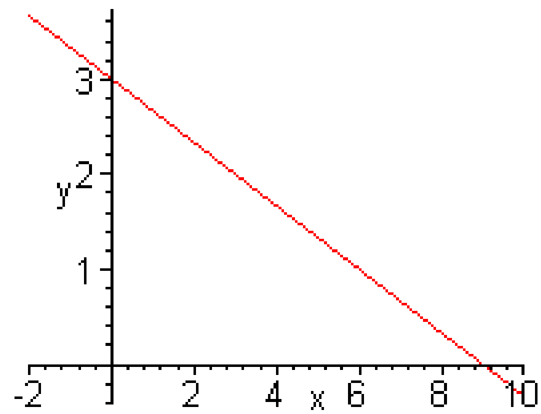




22. The line given can be written $y = \frac{3}{5}x - \frac{7}{5}$. Thus, its slope is $m = \frac{3}{5}$. A parallel line has the same slope, and passing through the origin, its y-intercept is 0. Thus, the desired line satisfies the equation $y = \frac{3}{5}x$. Above right is a graph of both lines.

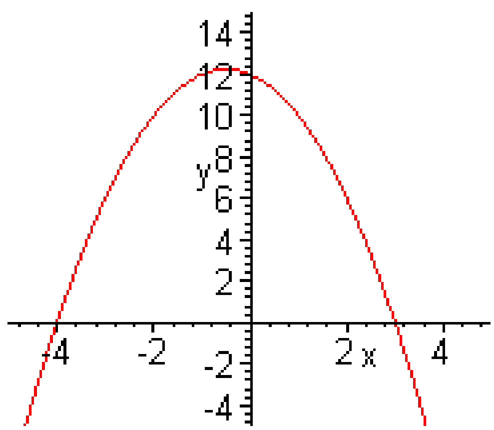
23. Since the slope of the given line ($y = 4 - 2x$) is -2, a perpendicular line has the negative reciprocal slope or $m = \frac{1}{2}$. Since it passes through the point $(1, 2)$, we have $2 = \frac{1}{2}(1) + b$ or $b = \frac{3}{2}$. Thus, the desired line is given by $y = \frac{1}{2}x + \frac{3}{2}$. Below left is a graph of both lines.

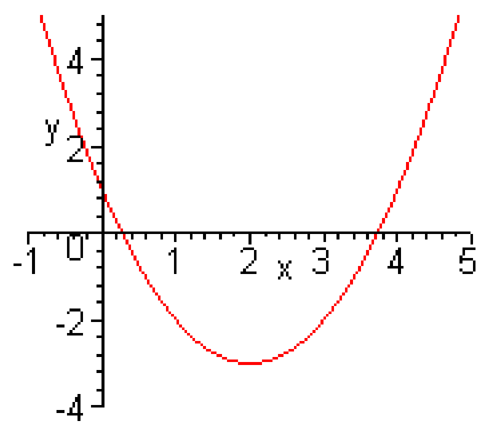




24. The desired line has the form $y = -\frac{1}{3}x + b$, so $4 = -\frac{1}{3}(-3) + b$, which gives $b = 3$. Thus, the line is given by $y = -\frac{1}{3}x + 3$. The x-intercept is $(9, 0)$, and the y-intercept is $(0, 3)$. Above right is a graph of the line.

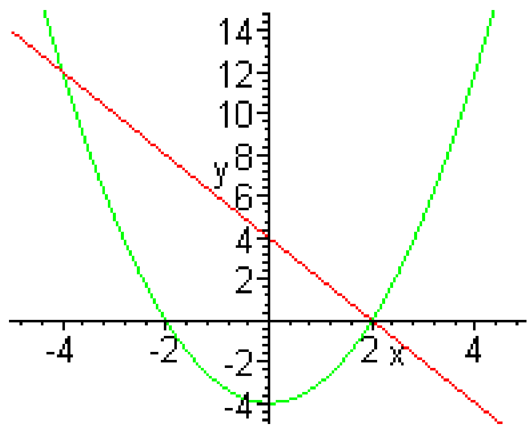
25. The y-intercept is found by setting $x = 0$, so occurs at $(0, 12)$. The x-intercepts satisfy $12 - x - x^2 = 0$, so $x^2 + x - 12 = (x + 4)(x - 3) = 0$. This gives the x-intercepts as $(-4, 0)$ and $(3, 0)$. The x value for the vertex is midway between the two x-intercepts, so $x = -\frac{1}{2}$. Solving for y, we have $y = 12 + \frac{1}{2} - \frac{1}{4} = 12\frac{1}{4}$. Thus, the vertex occurs at $(-\frac{1}{2}, 12\frac{1}{4})$. The graph of this parabola is shown below left.





26. The y-intercept is found by setting $x = 0$, so occurs at $(0, 1)$. The x-intercepts satisfy $x^2 - 4x + 1 = 0$, so by the quadratic formula the x-intercepts are $(2 + \sqrt{3}, 0)$ and $(2 - \sqrt{3}, 0)$. The x value for the vertex is midway between the two x-intercepts, so $x = 2$. Solving for y, we have $y = 4 - 8 + 1 = 3$. Thus, the vertex occurs at $(2, -3)$. The graph of this parabola is shown above right.

27. The x and y-intercepts for the line are $(2,0)$ and $(0,4)$, respectively. The x-intercepts for the quadratic are $(-2,0)$ and $(2,0)$, while the y-intercept is $(0,-4)$. The x-values for the points of intersection satisfy $4-2x = x^2-4$ or $x^2+2x-8 = (x+4)(x-2) = 0$, which gives $x=-4$ or 2. Using either equation to find the y value, the points of interesection are $(-4,12)$ and $(2,0)$. The graphs of both functions are shown below.



28. $\dfrac{(x+1)^3y^{-2}}{y(x+1)^2} = \dfrac{(x+1)^{3-2}}{y^{1-(-2)}} = \dfrac{x+1}{y^3}$.

29. $\dfrac{x^3y^2z^{-1}}{xy^{-2}z^2} = \dfrac{x^2y^4}{z^3}$.

30. $(9^{4/5})^{5/8} = 9^{4/8} = 9^{\frac{1}{2}} = 3$.

31. $(4^{-1/2})\left(\dfrac{1}{8}\right)^{-2/3} = \dfrac{1}{2}(2^{-3})^{-2/3} = 2$.

32. $\dfrac{(27)^{1/3}}{(\frac{1}{3})^{-1}} = \dfrac{(3^3)^{1/3}}{3} = 1$.

33. Since $f(x) = 2x-1$ and $g(x) = x^2+4$, we have $f(g(x)) = 2g(x)-1 = 2(x^2+4)-1 = 2x^2+7$.

34. Since $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x+3}$, $f(g(x)) = \frac{1}{g(x)} = \frac{1}{\sqrt{x+3}}$. Thus, $f(g(1)) = \frac{1}{\sqrt{1+3}} = \frac{1}{2}$. Similarly, $g(f(x)) = \sqrt{f(x)+3} = \sqrt{\frac{1}{x}+3}$, so $g(f(1)) = \sqrt{1+3} = 2$.

35. For $f(x) = \frac{2}{x+3}$, we have

$$\frac{f(1+h)-f(1)}{h} = \frac{\frac{2}{1+h+3} - \frac{2}{1+3}}{h} = \frac{1}{h}\left(\frac{2}{4+h} - \frac{2}{4}\right) = \frac{1}{h}\left(\frac{8-2(4+h)}{4(4+h)}\right) = \frac{1}{h}\left(\frac{-2h}{4(4+h)}\right) = -\frac{1}{2(4+h)}.$$

36. a. Since $P = P_0(1+r)^n$, it follows that $(1+r)^n = \frac{P}{P_0}$ or $1+r = \left(\frac{P}{P_0}\right)^{1/n}$. Thus, $r = \left(\frac{P}{P_0}\right)^{1/n} - 1$.

b. Since $P = P_0(1+r)^n$, it follows that $\log(P) = \log(P_0(1+r)^n) = \log(P_0) + n\log(1+r)$, so $n\log(1+r) = \log(P) - \log(P_0)$. Hence, $n = \frac{\log(P)-\log(P_0)}{\log(1+r)}$.

c. From the formula, $P = 10000(1+0.04)^2 = 10000(1.0816) = 10,816$, so the amount of capital after 2 years is \$10,816.

37. Let x be the length of one side, then $2x$ is the length of the over. The area is the product of the two sides, so $A = 2x^2 = 24$. Thus, $x^2 = 12$ and $x = 2\sqrt{3}$ cm. The other side is $4\sqrt{3}$ cm.

38. The area of a triangle is $\frac{1}{2}bh$, where b is the length of the base and h is the height of the triangle. The height h goes halfway between the two equal sides and is perpendicular to the base, $b = 6$, dividing it into 2 equal parts. Thus, half of the base, $a = 3$, and the height form a right triangle with its hypotenuse being the side of length, $c = 5$. By Pythagoras's Theorem we have $a^2 + h^2 = c^2$ or $9 + h^2 = 25$, which gives $h^2 = 16$ or $h = 4$. It follows that the a rea of the isoceles triangle is $A = \frac{1}{2}(6)(4) = 12$ cm^2.

39. If a population of 20,000 grows by 5%, then it is $20,000 \times 1.05 = 21,000$ at the end of the first year. In the second year, it again grows by 5%, which gives $21,000 \times 1.05 = 22,050$ at the end of the second year.

40. The monthly bill for electricity is now $\$85 \times 3 = \$255$.

41. For a \$28 dinner, the tip should be $\$28 \times 0.15 = \$4.20$. If you round to the nearest dollar, then the amount that you leave at the table would be $\$28 + \$4 = \$32$, but if you are polite and don't want to undertip, then you should leave \$33.

42. a. The travel time for Thelma is $30/75 = 0.4$ hours, which is $0.4(60) = 24$ min. Thus, she needs to leave at 9:36 am. The travel time for Louise is $60/60 = 1$ hour, so she must leave at 9:00 am.

b. The party consists of 2 adults and 7 children, so the total cost (undiscounted) is $2(\$38) + 7(\$32) = \$76 + \$224 = \$300$. The 10% discount is \$30, so the total cost is \$270.

43. The total rainfall for San Diego averages

$$1.8 + 1.53 + 1.77 + 0.79 + 0.19 + 0.07 + 0.02 + 0.1 + 0.24 + 0.37 + 1.45 + 1.57 = 9.9 \text{ inches.}$$

The mean (average) rainfall is $9.9/12 = 0.825$ inches/month.

44. a. The total rain received by the yard on average is $9.9 \times 50 \times 12 \times 200 \times 12 = 14,256,000$ in^3. In gallons, this is $14,256,000/230 = 61,983$ gallons.

b. The number of flushes of the toilet is $61{,}983/1.25 = 49{,}586$.

45. Let x be the number of 25 cent comic books, so $x+2$ is the number of 40 cent comic books. We have $0.25x + 0.40(x+2) = 4.70$ or $0.65x = 3.90$, which implies that $x = 6$. Thus, you buy six 25 cent comic books and eight 40 cent comic books.

46. This problem has two solutions (assuming the simplest case where the points are colinear, *i.e.*, all lie on a line). If the points are arranged with Point B between Point A and C, then if we let x be the distance between Points A and B, we have the distance between Points A and C is $2x$ and the distance from Points B and C is x. Thus, $x = 5$, so the distance between Points A and C is 10 inches.

If the points are arranged on a line with Point A between Points B and C, then if we let x be the distance between Points A and B, we have the distance between Points A and C is $2x$ and the distance from Points B and C is $3x$. Since $3x = 5$, we have $x = \frac{5}{3}$, which implies that the distance between Points A and C is $\frac{10}{3}$ inches.

47. The relative speed between the two vehicles is 75 mph, so they jointly cover the 50 miles in $\frac{50}{75} = \frac{2}{3}$ hours $= 40$ min. So you pass Mr. Jones at 5:40 pm (assuming not too much rush hour traffic). Not asked, but you can see that the cars pass each other $26\frac{2}{3}$ miles from your home.

48. Let x be the length of a side of this box. The open rectangular box has four sides each with an area of $x \times 1$ ft^2 and a bottom with an area of x^2 ft^2. It follows that the area of the box satisfies $x^2 + 4x = 12$ or $x^2 + 4x - 12 = (x+6)(x-2) = 0$. Since x cannot be -6, it follows that the bottom edge is 2 ft on a side. Thus, the box is 2 ft by 2 ft by 1 ft, which gives a volume of 4 ft^3.

49. Let x be the units of milk per day that a black cow gives and y be the units of milk per day that a brown cow gives. From the statement of the problem, $4(5x) + 3(5y) = V$, where V is the total volume after 5 days and $3(4x) + 5(4y) = V$. It follows that $20x + 15y = 12x + 20y$ or $8x = 5y$. So $x/y = 5/8$, which means that a black cow gives only 5/8 of the milk that a brown cow gives. Hence, the brown cow is the better milker.

50. Let x be the number of dimes, y be the number of quarters, and z be the amount of money that the man has. It follows that $x + y = 20$, $0.10x + 0.25y = z$, and $0.25x + 0.10y = z + 0.9$. The first equation gives $y = 20 - x$, which we substitute into the other equations giving $0.10x + 0.25(20 - x) = z$ and $0.25x + 0.10(20 - x) = z + 0.9$. It follows that $z + 0.15x = 5$ and $z - 0.15x = 1.1$. Subtracting the second equation from the first, gives $0.3x = 3.9$ or $x = 13$. It follows that $y = 7$ and $z = \$3.05$.