2. a. The height of the ball at $t = 0$ is $h(0) = 64$ ft, and the height at 2 seconds is $h(2) = -16(2)^2 + 48(2) + 64 = 96$ ft. The average velocity for $t \in [0, 2]$ is

$$v_{ave} = \frac{h(2) - h(0)}{2 - 0} = \frac{96 - 64}{2} = 16 \text{ ft/sec.}$$

At $t = 2.5$, the height is $h(2.5) = -16(2.5)^2 + 48(2.5) + 64 = 84$ ft. The average velocity for $t \in [2, 2.5]$ is

$$v_{ave} = \frac{h(2.5) - h(2)}{2.5 - 2}\frac{84 - 96}{0.5} = -24 \text{ ft/sec.}$$

b. The graph of the height of the ball is a parabola (shown below), so the maximum height occurs at the vertex of the parabola. The vertex of the parabola is midway between the two t-intercepts, *i.e.* where $h = 0$. Thus, $h(t) = -16t^2 + 48t + 64 = -16(t^2 - 3t - 4) = -16(t - 4)(t + 1) = 0$, when $t = -1$ and $t = 4$. The t-value of the vertex is halfway between, $t = 1.5$. It follows that the maximum height of the ball occurs at 1.5 sec with the height $h(1.5) = 100$ ft.

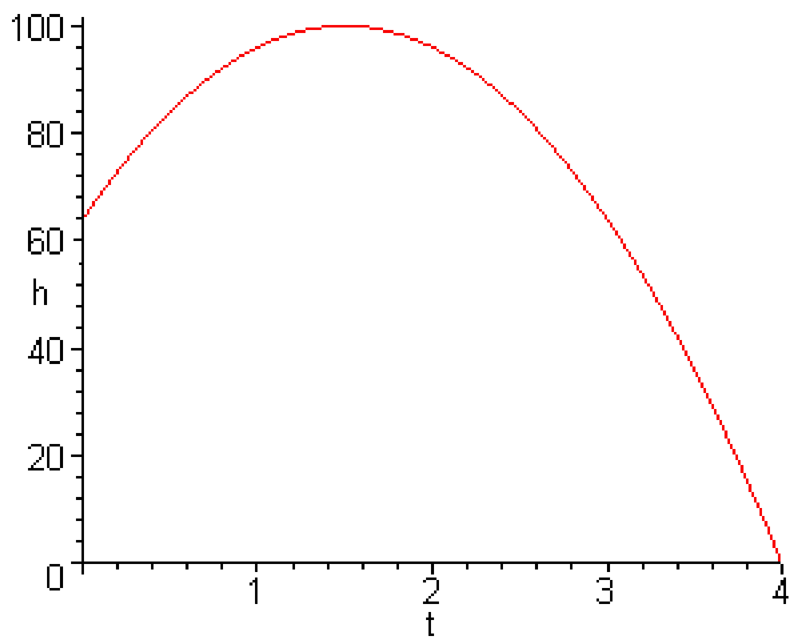
c. From above, we saw that the ball hits the ground at $t = 4$ sec. To find the velocity, consider a time $t = 4 + \Delta t$ and the height

$$\begin{aligned} h(4 + \Delta t) &= -16(4 + \Delta t)^2 + 48(4 + \Delta t) + 64 \\ &= -16(4)^2 - 16(8)\Delta t - 16\Delta t^2 + 48(4) + 48\Delta t + 64 \\ &= -80\Delta t - 16\Delta t^2. \end{aligned}$$

The velocity of the ball hitting the ground satisfies

$$v(4) = \frac{h(4 + \Delta t) - h(4)}{4 + \Delta t - 4} = \frac{-80\Delta t - 16\Delta t^2 - 0}{\Delta t} = -80 - 16\Delta t.$$

As Δt gets small, it is easy to see that this velocity goes to -80 ft/sec. Thus, the ball hits the ground with a velocity of -80 ft/sec.



4. a. The maximum height is between the two t-intercepts. We solve $h(t) = 12 + 16t - 16t^2 = -4(4t^2 - 4t - 3) = -4(2t + 1)(2t - 3) = 0$ or $t = -0.5$ and $t = 1.5$. Halfway between the intercepts is $t = 0.5$ with

$$h(0.5) = 12 + 16(0.5) - 16(0.5)^2 = 16.$$

Since the cat achieves its maximum height at $t = 0.5$ sec with a maximum height of 16 ft, the cat cannot reach the bird (at 18 ft).

b. Average velocity is found by determining the beginning and ending heights, then dividing the difference by the time difference, in this case 0.5 seconds. By associating the velocity with the earlier time, we have

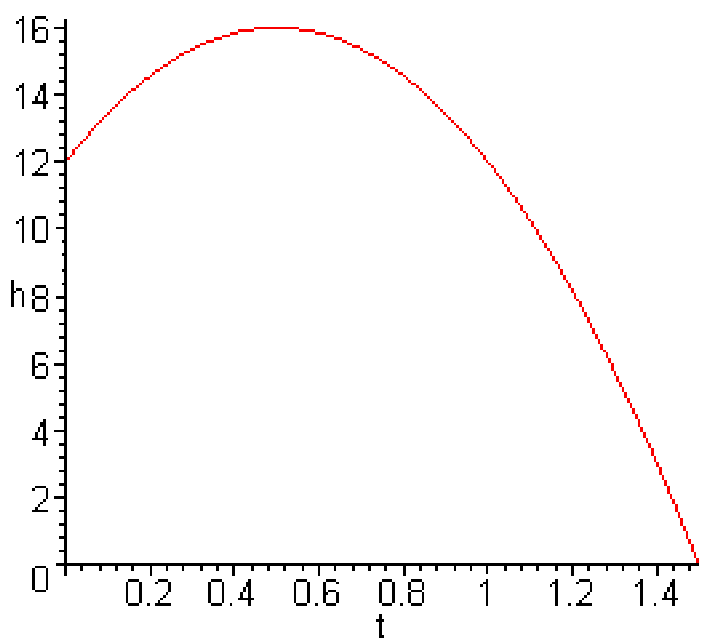
$$\begin{aligned} v(0) &= \frac{h(0.5) - h(0)}{0.5} = \frac{16 - 12}{0.5} = 8 \\ v(0.5) &= \frac{h(1) - h(0.5)}{0.5} = \frac{12 - 16}{0.5} = -8 \\ v(1) &= \frac{h(1.5) - h(1)}{0.5} = \frac{0 - 12}{0.5} = -24 \end{aligned}$$

Thus, the average velocity of the cat for the intervals $t \in [0, \frac{1}{2}]$ is 8 ft/sec, $t \in [\frac{1}{2}, 1]$ is -8 ft/sec, and $t \in [1, \frac{3}{2}]$ is -24 ft/sec.

c. The cat hits the ground when $h(t) = 0$ at time $t = 1.5$ sec. The velocity of impact is found by considering a time $t = 1.5+\Delta t$. The height is $h(1.5+\Delta t) = 12+16(1.5+\Delta t)-16(1.5+\Delta t)^2 = 0 - 32\Delta t - 16\Delta t^2$. It follows that the velocity at 1.5 is approximated by

$$v(1.5) = \frac{h(1.5 + \Delta t) - h(1.5)}{\Delta t} = \frac{-32\Delta t - 16\Delta t^2 - 0}{\Delta t} = -32 - 16\Delta.$$

For small Δt, this quantity converges to $v(1.5) = -32$ ft/sec. Thus, the cat hits the ground at $t = 1.5$ sec with a velocity of $v(1.5) = -32$ ft/sec. A sketch of the graph for the height of the cat as a function of t is below.



5. a. Since the kangaroo can jump 240 cm, this is the highest value of $h(t)$, which is the vertex of the quadratic function $h(t)$. First we find the t-intercepts for $h(t)$, which satisfy $h(t) = v_0 t - 490t^2 = t(v_0 - 490t) = 0$. It follows that either $t = 0$ or $t = \frac{v_0}{490}$. The t-value of the vertex is the midpoint, so $t = \frac{v_0}{980}$. It follows that the height of the vertex satisfies

$$h(v_0/980) = v_0\left(\frac{v_0}{980}\right) - 490\left(\frac{v_0}{980}\right)^2 = \frac{v_0^2}{1960} = 240.$$

So the initial velocity is given by $v_0 = \sqrt{1960(240)} = 280\sqrt{6} \simeq 685.9$ cm/sec. The kangaroo is in the air until $t = \frac{v_0}{490} = \frac{4}{7}\sqrt{6} \simeq 1.40$ sec.

b. The average velocity of the kangaroo between $t = 0$ and $t = 1$ is

$$v_{ave} = \frac{h(1) - h(0)}{1 - 0} = 280\sqrt{6} - 490 \simeq 195.9 \text{ cm/sec.}$$

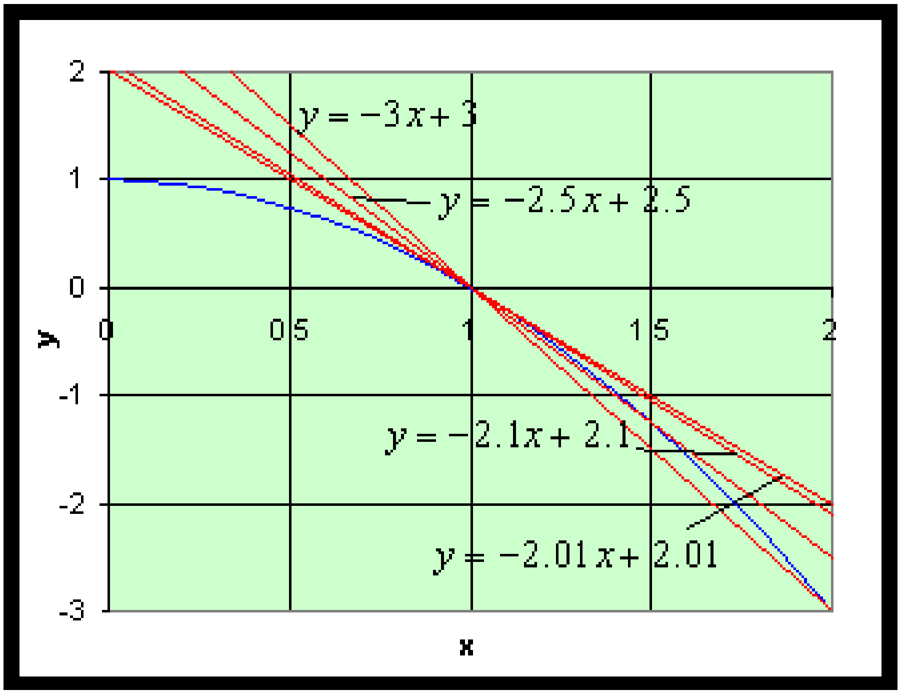
6. a. With the function $f(x) = 1 - x^2$, we are given the point $(1, 0)$ on the curve. We use the point slope form of the line to find each line in the desired sequence. First, we evaluate the function at each of the x-values listed, so $f(2) = 1 - 2^2 = -3$, $f(1.5) = 1 - 1.5^2 = -1.25$, $f(1.1) = 1 - 1.1^2 = -0.21$ and $f(1.01) = 1 - 1.01^2 = -0.0201$ The slopes of the secant lines satisfy:

$$\begin{aligned}
m &= \frac{f(2) - f(1)}{2 - 1} = \frac{-3 - 0}{1} = -3 \\
m &= \frac{f(1.5) - f(1)}{1.5 - 1} = \frac{-1.25 - 0}{0.5} = -2.5 \\
m &= \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{-0.21 - 0}{0.1} = -2.1 \\
m &= \frac{f(1.01) - f(1)}{1.01 - 1} = \frac{-0.0201 - 0}{0.01} = -2.01
\end{aligned}$$

To find the y-intercept b of the secant lines, we solve $0 = m(1) + b$, so $b = -m$. It follows that the sequence of secant lines are:

$$\begin{aligned}
y &= -3x + 3 \\
y &= -2.5x + 2.5 \\
y &= -2.1x + 2.1 \\
y &= -2.01x + 2.01
\end{aligned}$$

6. b. It is easy to see that the slopes of the secant lines are converging to $m = -2$, so the tangent line at $x = 1$ is $y = -2x + 2$. The slope at $x = 1$ is -2, so the derivative of $f(x)$ at $x = 1$ is -2. Below is a graph of the secant lines.



8. For each of the following problems, we compute the slope of the secant line by the formula

$$m = \frac{f(1 + \Delta x) - f(1)}{\Delta x}$$

and the intercept

$$b = f(1) - m.$$

a. The function satisfies $f(1) = 1^2 + 2 = 3$ and $f(1_+\Delta x) = (1 + \Delta x)^2 + 2 = 3 + 2\Delta x + \Delta x^2$. From the formulae above,

$$m = \frac{3 + 2\Delta x + \Delta x^2 - 3}{\Delta x} = 2 + \Delta x$$

and

$$b = 3 - (2 + \Delta x) = 1 - \Delta x.$$

The secant line is $y = (2 + \Delta x)x + 1 - \Delta x$. As $\Delta x \to 0$, the slope of the tangent line is given by $m_t = 2$.

8. c. The function satisfies $f(1) = \sqrt{2(1)+2} = 2$ and $f(1+\Delta x) = \sqrt{2(1+\Delta x)+2} = \sqrt{4+2\Delta x}$. From the formulae above,

$$m = \frac{\sqrt{4+2\Delta x}-2}{\Delta x} = \left(\frac{\sqrt{4+2\Delta x}-2}{\Delta x}\right)\left(\frac{\sqrt{4+2\Delta x}+2}{\sqrt{4+2\Delta x}+2}\right) = \frac{2}{\sqrt{4+2\Delta x}+2}$$

and

$$b = 2 - \frac{2}{\sqrt{4+2\Delta x}+2}.$$

The secant line is $y = \frac{2}{\sqrt{4+2\Delta x}+2}x + 2 - \frac{2}{\sqrt{4+2\Delta x}+2}$. As $\Delta x \to 0$, the slope of the tangent line is given by $m_t = \frac{1}{2}$.

e. The function satisfies $f(1) = \frac{1}{2}$ and $f(1_+\Delta x) = \frac{1}{2+\Delta x}$. From the formulae above,

$$m = \frac{\frac{1}{2+\Delta x} - \frac{1}{2}}{\Delta x} = \frac{2-(2+\Delta x)}{2\Delta x(2+\Delta x)} = -\frac{1}{2(2+\Delta x)}$$

and

$$b = \frac{1}{2} + \frac{1}{2(2+\Delta x)}.$$

The secant line is $y = -\frac{1}{2(2+\Delta x)}x + \frac{1}{2} + \frac{1}{2(2+\Delta x)}$. As $\Delta x \to 0$, the slope of the tangent line is given by $m_t = -\frac{1}{4}$.