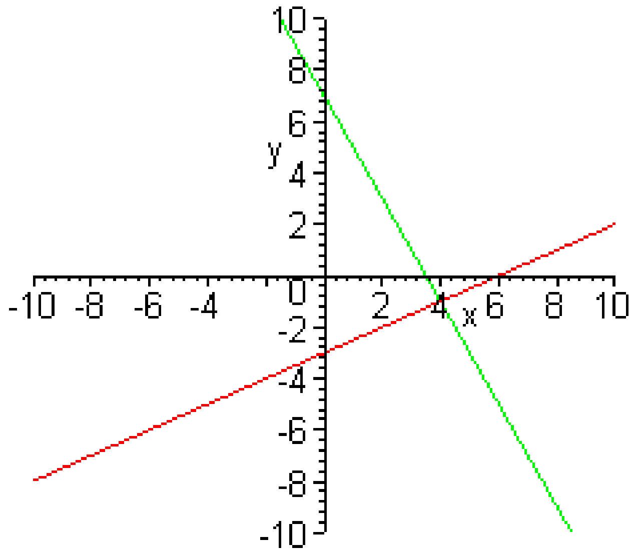
1. The slope is given by $m_1 = \frac{-1-(-3)}{4-0} = \frac{1}{2}$. The first point is the y-intercept, so the equation of the line is $y = \frac{x}{2} - 3$. The slope of the line perpendicular to this line is the negative reciprocal, which is $m_2 = -2$. (Note that $m_1 m_2 = -1$.) Using the point slope of the line, we have $y - (-1) = -2(x - 4)$, which simplifies to $y = -2x + 7$. The graph of these two lines is below.



2. Since 1 kg = 2.2046 lb, it follows that

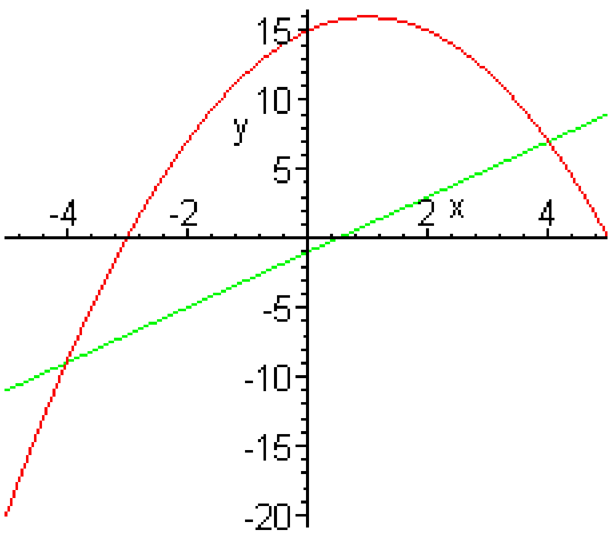
$$43 \text{ lb} \left(\frac{1 \text{ kg}}{2.2046 \text{ lb}} \right) = 19.5 \text{ kg}.$$

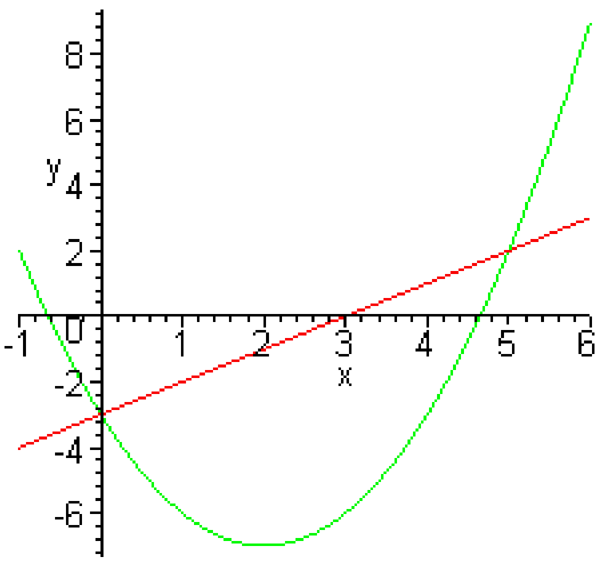
The temperature conversion from Fahrenheit to Celsius is $c = \frac{5}{9}(f - 32)$, so

$$c = \frac{5}{9}(102 - 32) = 38.9°\text{C}.$$

3. For the line, $f(x) = 2x - 1$, the y-intercept occurs at $(0, -1)$, while the x-intercept satisfies $2x - 1 = 0$ or $x = \frac{1}{2}$. The slope is $m = 2$. For $g(x) = 15 + 2x - x^2$, the y-intercept satisfies $g(0) = 15$, while the x-intercepts satisfy $x^2 - 2x - 15 = (x - 5)(x + 3) = 0$, so $x = 5, -3$. The x value of the vertex is the midpoint between the x-intercepts, so $x = 1$ and $g(1) = 16$, so the vertex is $(1, 16)$.

The curves intersect when $f(x) = g(x)$ or $15 + 2x - x^2 = 2x - 1$. Thus, $x^2 - 16 = (x+4)(x-4) = 0$, which implies that $x = \pm 4$. Since $g(4) = f(4) = 7$ and $g(-4) = f(-4) = -9$, the points of intersection are $(-4, -9)$ and $(4, 7)$. The graph of the line and the parabola are below to the left.





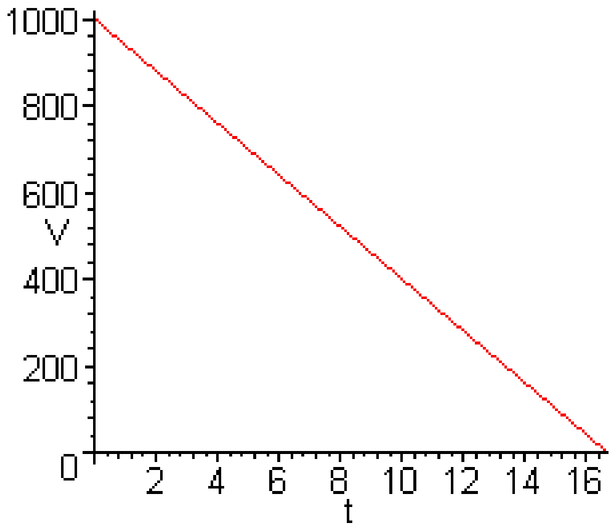
4. For the line, $f(x) = x - 3$, the y-intercept occurs at $(0, -3)$, while the x-intercept satisfies $x - 3 = 0$ or $x = 3$. The slope is $m = 1$. For $g(x) = x^2 - 4x - 3$, the y-intercept satisfies $g(0) = -3$. The x-intercepts satisfy $x^2 - 4x - 3 = 0$, which by the quadratic formula are given by

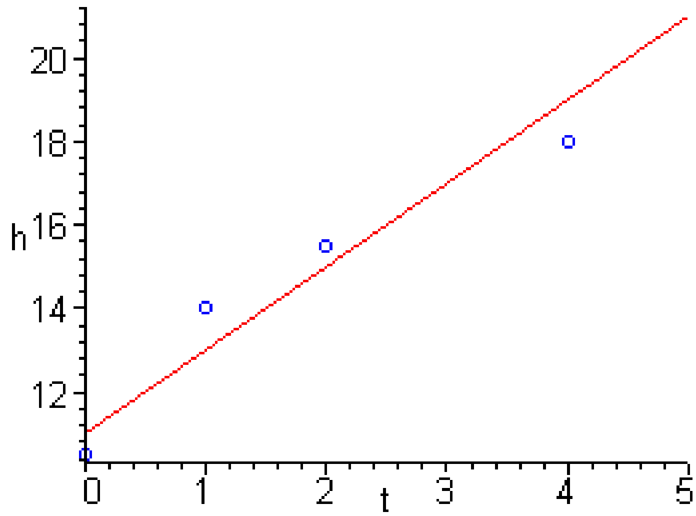
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2} = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{7}.$$

The x value of the vertex is the midpoint between the x-intercepts, so $x = 2$ and $g(2) = -7$, so the vertex is $(2, -7)$.

The curves intersect when $f(x) = g(x)$ or $x^2 - 4x - 3 = x - 3$. Thus, $x^2 - 5x = x(x - 5) = 0$, which implies that $x = 0$ or $x = 5$. Since $g(0) = f(0) = -3$ and $g(5) = f(5) = 2$, the points of intersection are $(0, -3)$ and $(5, 2)$. The graph of the line and the parabola are above to the right.

5. The data lie on a line, so the linear model satisfies $V = mt + b$. The slope is given by $m = \frac{940-1000}{1-0} = -60$. Clearly, the V-intercept is 1000, so the linear model is given by $V = 1000 - 60t$. The water is lost when $V(t) = 0 = 1000 - 60t$ or at time $t = \frac{1000}{60} \simeq 16.7$ weeks. Note that this model is only valid for $0 \le t \le \frac{100}{6}$. The graph of the water evaporating is shown below to the left.





6. a. The linear model for the height of a plant is given as $h(t) = 2t + 11$, while growth data is

Week (t)	0	1	2	4
Height (cm) (h)	10.5	14	15.5	18

The error between the data and the model at each time is given by

$$\begin{aligned} e_1 &= |10.5 - h(0)| = |10.5 - (2(0) + 11)| = 0.5 \\ e_2 &= |14 - h(1)| = |14 - (2(1) + 11)| = 1 \\ e_3 &= |15.5 - h(2)| = |15.5 - (2(2) + 11)| = 0.5 \\ e_4 &= |18 - h(4)| = |18 - (2(4) + 11)| = 1 \end{aligned}$$

The sum of the squared error is given by $J = e_1^2 + e_2^2 + e_3^2 + e_4^2 = 0.25 + 1 + 0.25 + 1 = 2.5$. The graph of the model with the data points is above to the right. Over the given domain the model is reasonable. However, this model predicts that the plant will keep growing at a constant rate, which is not commonly observed.

b. The model predicts the height of the plant at 3 and 5 weeks to be

$$\begin{aligned} h(3) &= 2(3) + 11 = 17 \text{ cm} \\ h(5) &= 2(5) + 11 = 21 \text{ cm} \end{aligned}$$

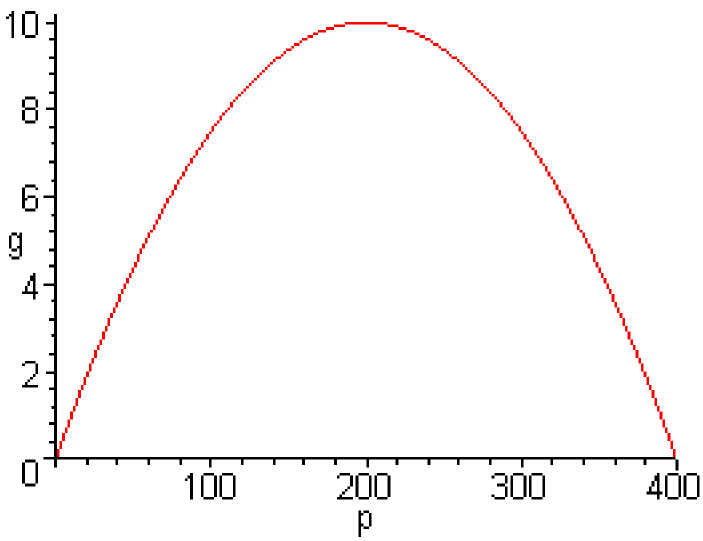
7. a. The equilibrium population satisfies

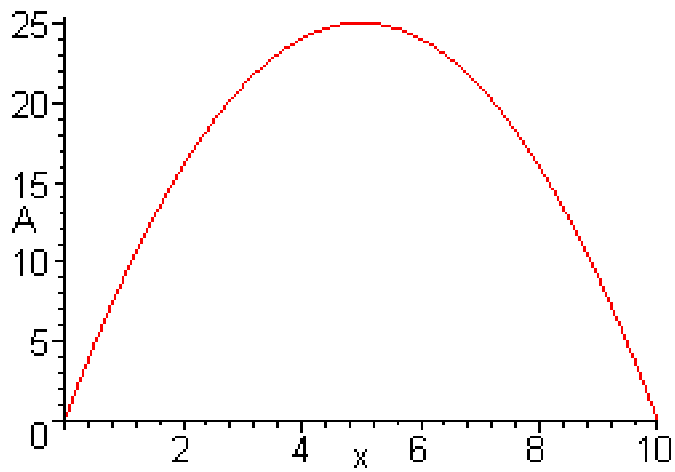
$$g(P_e) = 0.1P_e\left(1 - \frac{P_e}{400}\right) = 0,$$

which is in factored form, so $P_e = 0$ or $1 - P_e/400 = 0$, which gives $P_e = 400$ individuals. Thus, the equilibrium populations are $P_e = 0$ and 400.

b. The growth function $g(p)$ graphs a parabola with the equilibrium populations forming the intercepts. The vertex of parabola is the midpoint between the P-intercepts of the growth function, so occurs at the population $P = 200$. The maximum growth rate is $g(200) = 0.1(200)\left(1 - \frac{200}{400}\right) = 10$ individuals per generation.

c. The graph $g(P) = 0.1P(1 - \frac{P}{400})$ is shown below to the left.



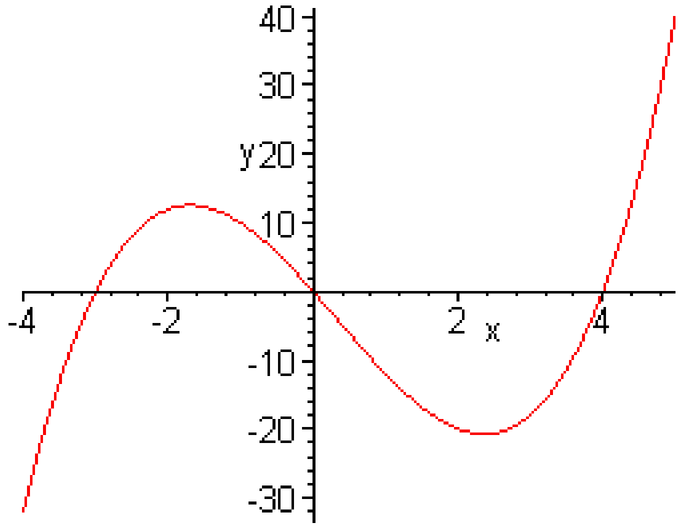


8. a. The perimeter of the rectangle is 20 cm, so $2y + 2x = 20$ or $2y = 20 - 2x$. It follows that $y = 10 - x$.

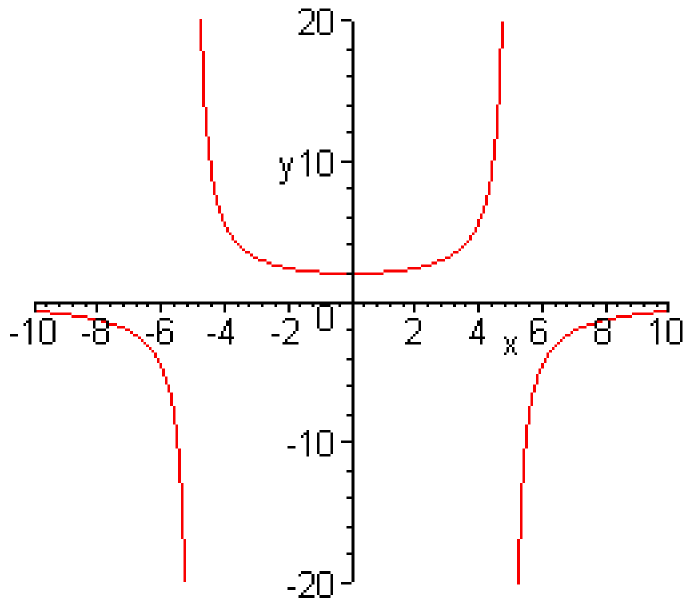
b. The area of a rectangle is $A = xy$, so $A(x) = x(10 - x) = 10x - x^2$. Since $A(x) \geq 0$ it follows that $0 \leq x \leq 10$. (This is easily seen from the x-intercepts being $x = 0$ and 10.

c. $A(x)$ is a parabola, and its graph is shown above to the right. The vertex occurs at $x = 5$ with $A(5) = 25$ cm^2. Thus, the particular rectangle with the largest area is a square that measures 5 cm on a side.

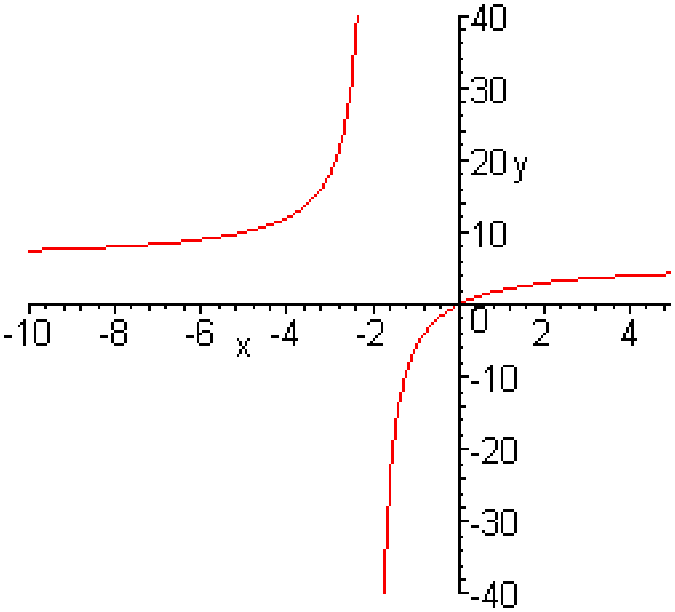
9. a. For $y = x^3 - x^2 - 12x$, the domain is all x. The x-intercept satisfies $y = 0 = x(x^2 - x - 12) = x(x-4)(x+3)$. So the x-intercepts are 0, 4 and -3. The y-intercept is where $x = 0$, so the y-intercept is $y = 0$. There are no vertical or horizontal asymptotes. The graph is below.

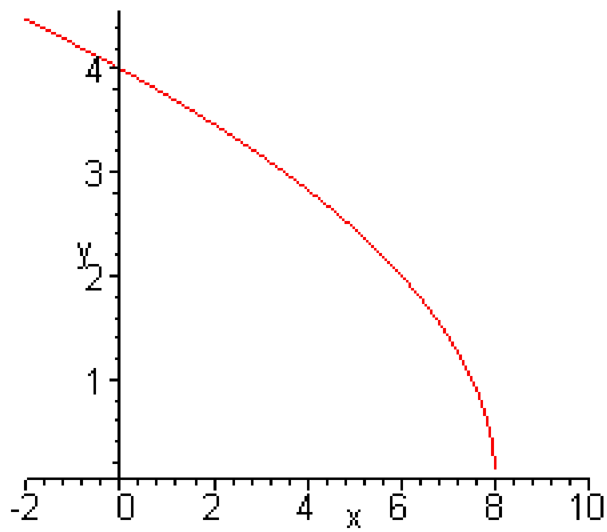


9. b. For $y = \dfrac{50}{25 - x^2}$, the domain is determined by finding where the denominator is zero or $x^2 - 25 = (x+5)(x-5) = 0$, so the domain is $x \neq \pm 5$. The x-intercept satisfies $y = 0$. Since the numerator is never zero, there is no x-intercept. The y-intercept is where $x = 0$, so the y-intercept is $y = \frac{50}{25} = 2$. The vertical asymptote occurs where the denominator is zero (boundary of the domain). Thus, $x^2 - 25 = 0$, gives vertical asymptotes of $x = \pm 5$. The horizontal asymptote is found using large x. Taking the highest powers in the numerator and denominator gives $y = \frac{1}{x^2}$, so the horizontal asymptote is $y = 0$. The graph is below.



9. c. For $y = \dfrac{6x}{x+2}$, the domain is determined by finding where the denominator is zero or $x + 2 = 0$, so the domain is $x \neq -2$. The x-intercept satisfies $y = 0$, which is when the numerator is zero. So $6x = 0$, which gives the x-intercept $x = 0$. The y-intercept is where $x = 0$, so the y-intercept is $y = 0$. The vertical asymptote occurs where the denominator is zero (boundary of the domain). Thus, $x + 2 = 0$, gives the vertical asymptote of $x = -2$. The horizontal asymptote is found using large x. Taking the highest powers in the numerator and denominator gives $y = \frac{6x}{x} = 6$, so the horizontal asymptote is $y = 6$. The graph is below to the left:





9. d. For $y = \sqrt{16 - 2x}$, the domain is determined by finding where the quantity under the radical is non-negative or $16 - 2x \geq 0$, so the domain is $x \leq 8$. The x-intercept satisfies $y = 0$, which is when $\sqrt{16 - 2x} = 0$, which gives the x-intercept $x = 8$. The y-intercept is where $x = 0$, so $y = \sqrt{16} = 4$. There are no vertical or horizontal asymptotes. The graph is above to the right.

10. a. If $e^a = 2.2$ and $e^b = 0.7$, then

$$\begin{aligned} \frac{e^{a+b}}{(e^b + e^0)^2} &= \frac{e^a e^b}{(e^b + 1)^2} \\ &= \frac{2.2(0.7)}{(0.7 + 1)^2} = \frac{1.54}{2.89} \simeq 0.5329. \end{aligned}$$

10. b. If $e^a = 2.2$ and $e^b = 0.7$, then

$$\begin{aligned} \frac{(e^a)^2(e^0 - e^b)}{e^{2a+b}} &= \frac{e^{2a}(1 - e^b)}{e^{2a} e^b} \\ &= \frac{1 - e^b}{e^b} \\ &= \frac{1 - 0.7}{0.7} = \frac{3}{7} \simeq 0.429. \end{aligned}$$

10. c. If $\ln(c) = 1.3$ and $\ln(d) = -0.5$, then

$$\begin{aligned}\frac{\ln(c^3) - \ln(c) + \ln(1)}{(\ln(c) + \ln(e))} &= \frac{3\ln(c) - \ln(c) + 0}{\ln(c) + 1} \\ &= \frac{2\ln(c)}{\ln(c) + 1} \\ &= \frac{2(1.3)}{1.3 + 1} = \frac{26}{23} \simeq 1.13.\end{aligned}$$

10. d. If $\ln(c) = 1.3$ and $\ln(d) = -0.5$, then

$$\begin{aligned}\frac{\ln(c^2 d) - \ln(1)}{(\ln(c/d) - \ln(e))} &= \frac{\ln(c^2) + \ln(d) - 0}{\ln(c) - \ln(d) - 1} \\ &= \frac{2\ln(c) + \ln(d)}{\ln(c) - \ln(d) - 1} \\ &= \frac{2(1.3) - 0.5}{1.3 - (-0.5) - 1} = \frac{2.1}{0.8} \simeq 2.625.\end{aligned}$$

11. a. Consider the linear model $F = mW + b$. The slope is given by $m = \frac{(1210-390)}{(2520-560} = 0.4184$. The intercept with one of the data points satisfies $b = 390 - 0.4184(560) = 155.7$, so the linear model is given by

$$F = 0.4184W + 155.7.$$

b. The allometric model satisfies:

$$\begin{aligned}F &= kW^a, \\ \ln(F) &= a\ln(W) + \ln(k).\end{aligned}$$

Note that if $Y = \ln(F)$, $X = \ln(W)$, and $K = \ln(k)$, then this is just the equation of a line

$$Y = aX + K,$$

where X and Y are the logarithms of the data.
The data given are shown with their logarithmic values in the table below:

W	$\ln(W)$	F	$\ln(F)$
560	6.328	390	5.966
2520	7.832	1210	7.098

From the formula above, the slope is a and satisfies

$$a = \frac{\ln(F_2) - \ln(F_1)}{\ln(W_2) - \ln(W_1)} = \frac{7.098 - 5.966}{7.832 - 6.328} = 0.7528.$$

To obtain k, we see that

$$\ln(k) = \ln(F_1) - a\ln(W_1) = 5.966 - 0.7528(6.328) = 1.2026,$$

so

$$k = e^{\ln(k)} = e^{1.2026} = 3.329.$$

This gives the allometric model

$$F = 3.329W^{0.7528}.$$

c. A 1000 g chicken consumes $F(1000) = 0.4184(1000) + 155.7 = 574.1$ g of feed according to the linear model and $F(1000) = 3.3329(1000)^{0.7528} = 603.4$ g.

If a chicken consumes 500 g of food, then the linear model gives

$$\begin{aligned} 500 &= 0.4184W + 155.7 \\ W &= \frac{500 - 155.7}{0.4184} = 822.9 \text{ g.} \end{aligned}$$

If a chicken consumes 500 g of food, then the allometric model gives

$$\begin{aligned} 500 &= 3.329W^{0.7528} \\ W &= \left(\frac{500}{3.329}\right)^{\frac{1}{0.7528}} = 779.0 \text{ g.} \end{aligned}$$

The allometric model is superior as the linear model would suggest that a non-existent chicken ($W = 0$ g) would still eat 155.7 g of feed.

12. a. The proposed model for relating length of a dog L given the height of a dog H is given by the linear model $L(H) = 1.7H + 10$. The errors are computed as follows:

$$\begin{aligned} e_1 &= |33 - (1.7(18) + 10)| = 7.6 \\ e_2 &= |81 - (1.7(36) + 10)| = 9.8 \\ e_3 &= |102 - (1.7(55) + 10)| = 1.5 \\ e_4 &= |115 - (1.7(66) + 10)| = 7.2 \end{aligned}$$

Sum of squares of the errors is $J = 7.6^2 + 9.8^2 + 1.5^2 + 7.2^2 = 207.9$. The beagle is furthest from the model.

b. The length of a Borzoi is predicted to be $L(81) = 1.7(81) + 10 = 148$ cm. Since $85 = 1.7H + 10$ gives $H = \frac{75}{1.7} = 44$ cm, the model suggests that the Border Collie will have a height of 44 cm.

c. The Border Collie is probably a better prediction because the data point lies within other data. If one were to use an allometric model, then the best power law model should be $L = a_1 H$, since both length and height are one-dimensional (with L expanding directly in proportion to H).

13. a. The linear model $T = mv + b$ has a slope given by

$$m = \frac{4-3}{0.45-0.25} = 5.$$

The intercept satisfies

$$b = 3 - 5(0.25) = 1.75.$$

Thus, the linear model is $T = 5v + 1.75$.

A 2 mm tissue sample solves $2 = 5v + 1.75$, so $v = \frac{0.25}{5} = 0.05$ V. If the experiment reveals a 0.6 V drop, then the tissue thickness is $T(0.6) = 5(0.6) + 1.75 = 4.75$ mm.

b. The allometric model satisfies:

$$\begin{aligned} T &= kv^a, \\ \ln(T) &= a\ln(v) + \ln(k). \end{aligned}$$

Note that if $Y = \ln(T)$, $X = \ln(v)$, and $K = \ln(k)$, then this is just the equation of a line

$$Y = aX + K,$$

where X and Y are the logarithms of the data.
The data given are shown with their logarithmic values in the table below:

v	$\ln(v)$	T	$\ln(T)$
0.25	-1.386	3	1.099
0.45	-0.799	4	1.386

From the formula above, the slope is a and satisfies

$$a = \frac{\ln(T_2) - \ln(T_1)}{\ln(v_2) - \ln(v_1)} = \frac{1.386 - 1.091}{-0.799 + 1.386} = 0.4894.$$

To obtain k, we see that

$$\ln(k) = \ln(T_1) - a\ln(v_1) = 1.099 - 0.5(-1.386) = 1.7771,$$

so

$$k = e^{\ln(k)} = e^{1.7771} = 5.913.$$

This gives the allometric model

$$T = 5.913V^{0.4894}.$$

If the thickness is 2 mm, then $2 = 5.913v^{0.4894}$, so $v = \left(\frac{2}{5.913}\right)^{1/0.4894} = 0.1092$V. If the voltage drop is 0.6V, then the thickness of the unknown tissue is $T(0.6) = 5.913(0.6)^{0.4894} =$ 4.605mm

c. The allometric model is superior because the voltage drop goes to zero when the thickness goes to zero.

14. a. From the data on the urea, the sum of squares of error is computed for the model $A = mc$

$$\begin{aligned} J(m) &= e_1^2 + e_2^2 + e_3^2 = (0.5 - 1m)^2 + (1.7 - 3m)^2 + (3.2 - 6m)^2 \\ &= 0.25 - m + m^2 + 2.89 - 10.2m + 9m^2 + 10.24 - 38.4m + 36m^2 \\ &= 46m^2 - 49.6m + 13.38 \end{aligned}$$

The m-value of the vertex is given by $m = 49.6/(2(46)) = 0.5391$. From this, we substitute into the sum of squares function $J(m)$ to obtain the least sum of squares

$$J(0.5391) = 46(0.5391)^2 - 49.6(0.5391) + 13.38 = 0.011.$$

b. Using the best slope in the model, $A = 0.5391c$, so $2.2 = 0.5391c$. This gives the concentration of the unknown urea sample as $c = \frac{2.2}{0.5391} = 4.08$ mM.

15. a. From the data in the photograph with the model $d = kp$, the sum of squares error satisfies

$$\begin{aligned} J(k) &= e_1^2 + e_2^2 + e_3^2 = (1.2 - 2k)^2 + (2.5 - 4k)^2 + (3.6 - 6.1k)^2 \\ &= 1.44 - 4.8k + 4k^2 + 6.25 - 20k + 16k^2 + 12.96 - 43.92k + 37.21k^2 \\ &= 57.21k^2 - 68.72k + 20.65 \end{aligned}$$

The k-value of the vertex is given by $k = 68.72/(2(57.21)) = 0.6006$. We substitute into the sum of squares function $J(k)$ to obtain the least sum of squares

$$J(0.6006) = 57.21(0.6006)^2 - 68.72(0.6006) + 20.65 = 0.0136$$

b. Using the best slope in the model, $d = 0.6006p$, the leopard shark measuring 2.2 cm, so $d = 0.6(2.2) = 1.32$ m. A 2 m shark would satisfy $2 = 0.6p$, so the size on the photograph would be $p = \frac{2}{0.6} = 3.3$ cm.