1. For each of the following functions, give the domain. Find the x and y-intercepts, and determine all vertical and horizontal asymptotes for each of these functions, then sketch a graph.

a. $y = 20 - 5e^{-0.5x}$,

b. $y = 6\ln(5 - x) - 2$,

c. $y = 3x - 2x^2 - x^3$,

d. $y = 4 - \sqrt{5 - x}$,

e. $y = \dfrac{4x}{2 + 0.001x}$,

f. $y = 6e^{x/2} - 2$,

g. $y = 3 + 2\ln(x + 1)$,

h. $y = \dfrac{8x}{4 - x^2}$.

2. a. A population of herbivores satisfies the growth equation

$$H_{n+1} = 1.02H_n.$$

If the initial population is $H_0 = 2000$, then determine the populations H_1 and H_2. Also, give an expression for the population H_n in terms of H_0 and n.

b. Another group of herbivores satisfies the growth equation

$$G_{n+1} = 1.03G_n.$$

If the initial population is $G_0 = 200$, then give an expression for the population G_n in terms of G_0 and n. Determine how long does it take for this population to double.

c. Find when the two populations are equal.

3. The population of the United States was about 179.3 million in 1960 and 226.5 million in 1980. Let 1960 be represented by P_0 and assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is in years.

a. Use the data above to find the annual growth rate r, then write an expression for the population in any year following 1960. (Write the solution P_n in terms of P_0 with n being the number of years after 1960.)

b. Predict the population in the year 2000. The actual population was about 281.4 million. What is the error between the model and the actual census data?

c. According to the model, how long until the U. S. population doubles from its 1960 level?

4. a. The population of the France in 1980 was about 53.9 million, and a census in 1990 showed that the population had grown to 56.7 million. Assume that this population grows according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980, and P_n is population n decades after 1980. Use the data above to find the growth constant r, then write the general solution P_n.

b. Predict the population in the years 2000 and 2020. France's population in 2000 was 59.4 million. Use this information to compute the percent error between the Malthusian growth model and the actual census data.

c. In 1980, the population of Kenya was 16.7 million, while in 1990, it had grown to 24.2 million. Assume its population is also growing according to a Malthusian growth law. Find its rate of growth per decade and predict its population in 2000 and 2020. How long does it take for Kenya's population to double?

d. If these countries continue to grow according to these Malthusian growth laws, then determine the first year when Kenya's population will exceed that of the France and determine their populations at that time.

e. Find the annual growth rate for both France and Kenya between 1980 and 1990.

5. A population of crustaceans living in a pond is affected by a pollutant that is slowly seeping into the ecosystem from an inflowing stream. The population, P_n, in organisms/liter after n weeks satisfies a nonautonomous Malthusian growth model

$$P_{n+1} = (1 + k(t_n))P_n \quad \text{with} \quad P_0 = 5,000.$$

where $k(t) = 0.12 - 0.03t$ is the growth rate of this invertebrate, which is clearly declining as t increases.

a. Find the population for this organism for the first 5 weeks.

b. When the growth rate falls to zero, this population reaches its maximum. Find when this occurs and what the population is at that time.

c. Determine when the pollution level gets so high that the crustaceans go extinct.

6. Suppose that a population of yeast in a culture satisfies the discrete logistic growth model

$$P_{n+1} = P_n + g(P_n),$$

where P_n is the population in (thousand/cm^3), $g(P)$ (individuals$\times$1000/cm^3/hr) is the growth rate of the population, and n is the number of hours. Assume that $g(P)$ has the quadratic form

$$g(P) = 0.15P\left(1 - \frac{P}{4000}\right).$$

a. Let $P_0 = 2000$ and compute P_1 and P_2.

b. Find when the growth rate, $g(P)$, is zero and when it is at a maximum. Find the population that produces this maximum growth rate and what that growth rate is, then sketch a graph of this growth rate function.

c. Find all equilibria for this model.

7. a. A population of herbivores reproduces annually and satisfies the discrete dynamical system:

$$P_{n+1} = P_n + rP_n,$$

where $r = 0.1$ is the net growth rate. If the initial population is $P_0 = 100$ individuals, then determine how many herbivores there are in each of the next two years. How long does it take for this population to double?

b. Often there are crowding effects due to limited resources. This is often modeled by the discrete logistic growth model. Assume these herbivores satisfy the model:

$$P_{n+1} = 1.1P_n - 0.0005P_n^2.$$

If the initial population is $P_0 = 100$, then find P_1 and P_2.

c. Find both equilibria for the discrete logistic equation in Part b.

8. Consider the discrete logistic growth model given by

$$p_{n+1} = f(p_n) = \frac{4}{3}p_n - \frac{1}{3000}p_n^2.$$

a. Suppose that the initial population p_0 is 100. Find the populations of the next two generations, p_1 and p_2.

b. Sketch a graph of the updating function, f, labeling any intercepts and the vertex. Also, sketch the identity map, $p_{n+1} = p_n$ and determine the intersection of these curves.

c. Find all equilibria and determine the stability of the equilibria (based on whether the iterations above move toward or away from the 2 equilibria).

9. a. Consider a model with immigration given by

$$p_{n+1} = 0.8p_n + 300,$$

with an initial population of $p_0 = 500$. Determine the populations at the next three time intervals, p_1, p_2, and p_3.

b. Find all equilibria and determine the stability of these equilibria.

10. A man with a chronic lung problem has a tidal volume, V_i, of 300 ml. For this experiment, Helium, He, is used to determine the functional reserve capacity, V_r. (Note that $V_r = (1-q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1-q)c_n + q\gamma,$$

where $\gamma = 5.2$ ppm.

a. The man is given an enriched mixture of air to breathe that contains 500 ppm of He. Experimentally, the concentration of He in the first two measured breaths after breathing the enriched mixture are given by $c_0 = 50$ and $c_1 = 44.6$ ppm. Use c_0 and c_1 to find q, then determine the functional reserve capacity, V_r.

b. Use your model to find the expected concentration of Helium in this patient's 3^{rd} breath, c_3. What is the equilibrium concentration of Helium in the patient's lungs? What is the stability of this equilibrium concentration?

11. Below are data on the population of insect pests living in a survey area. The insect reproduces according to a Malthusian growth model and disperses (emigrates) to surrounding regions at a constant rate. The population model for this insect pest is given by

$$P_{n+1} = (1+r)P_n - \mu,$$

where r is the rate of growth (per week) and μ is the number of pests dispersing each week to surrounding regions.

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of the insect pests for weeks 3 and 4.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Week	Insects
0	500
1	630
2	825

12. Consider Hassell's model that is given by

$$p_{n+1} = H(p_n) = \frac{6p_n}{1 + 0.001p_n}.$$

a. Assume that $p_0 = 2000$ and find the population for the next two generations, p_1 and p_2.

b. By solving $p_e = H(p_e)$, determine all equilibria for this model.

c. Find the p-intercepts and the horizontal asymptote for $H(p)$. Sketch a graph of $H(p)$ for $p > 0$ along with the identity map, $p_{n+1} = p_n$ and note where these functions intersect.

13. A selection experiment is done comparing two bacteria with different growth rates. The bacteria satisfy the Malthusian growth equations: $a_{n+1} = 1.8a_n$ and $b_{n+1} = 1.3b_n$. The fraction of bacteria (p_n) that are type a satisfies the equation:

$$p_{n+1} = \frac{1.8p_n}{1.3 + 0.5p_n}, \qquad 0 \le p_n \le 1,$$

where n is in hours.

a. If $a_0 = 50,000$, then find the population of type a bacteria after 5 hours. Write the general solution for the population of type a bacteria for any time n, assuming it continues to grow according to the Malthusian growth law given above.

b. Suppose that in the first generation it is found that $p_1 = 0.5$. Determine the fraction of bacteria of type a in the next generation and from the previous generation, *i.e.*, calculate p_2 and p_0.

c. Find the equilibrium fractions of type a bacteria from the model given above. What is the limiting fraction p_n for n large?

d. Sketch a graph of the updating function for the fraction of bacteria, p_n, of type a and the identity map for $0 \le p \le 1$.