3. We rewrite $f(x) = \dfrac{\sqrt{x}}{2+x} - \dfrac{1}{e^{3x}} = \dfrac{x^{1/2}}{2+x} - e^{-3x}$. Applying the quotient rule to the first and standard differentiation of exponentials to the second, we have:

$$f'(x) = \frac{(2+x)\frac{1}{2}x^{-1/2} - x^{1/2}(1)}{(2+x)^2} + 3e^{-3x}.$$

4. For $f(x) = \dfrac{x^2+5}{x^2-e^x} - \dfrac{xe^{2x}}{2x+1}$, we apply the quotient rule twice using the product rule in the numerator of the second expression.

$$f'(x) = \frac{2x(x^2-e^x) - (x^2+5)(2x-e^x)}{(x^2-e^x)^2} - \frac{(2x+1)(2xe^{2x}+e^{2x}) - 2xe^{2x}}{(2x+1)^2}.$$

6. $y = \dfrac{e^x}{x+1}$,

Derivative: By the quotient rule, $y' = \dfrac{(x+1)e^x - e^x(1)}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$.

Domain: $x \neq -1$.

y-intercept: $y(0) = 1$, so $(0, 1)$.

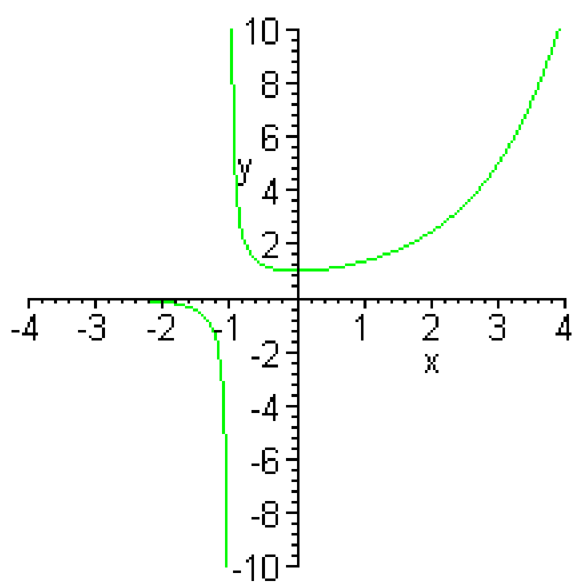
x-intercept: Since the exponential function is not zero, there are no x-intercepts.

Horizontal asymptote: As $x \to -\infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the left).

Vertical asymptote: Since the denominator $x + 1 = 0$ at $x = -1$, this is a vertical asymptote.

Critical points satisfy $y'(x) = 0$, so $xe^x = 0$ or $x = 0$. With $y(0) = 1$, $(0, 1)$ is a minimum.

The graph is below.



7. $y = \dfrac{x^2 - 2x + 2}{x - 1}$,

Derivative: By the quotient rule, $y' = \dfrac{(x-1)(2x-2) - (x^2 - 2x + 2)(1)}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$.

Domain: $x \neq 1$.

y-intercept: $y(0) = -2$, so $(0, -2)$.

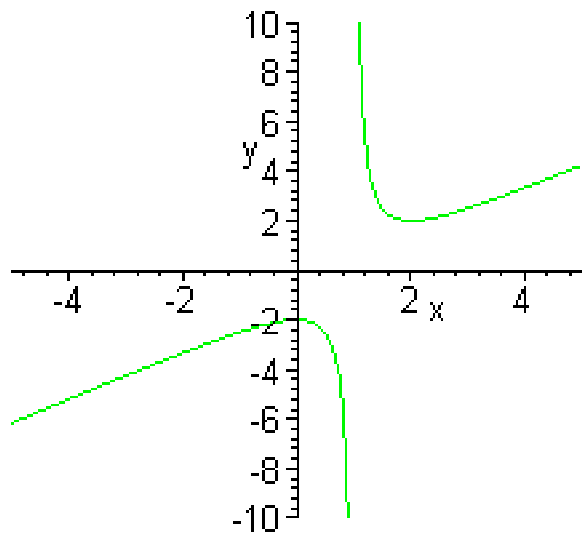
x-intercept: Since $x^2 - 2x + 2 = 0$ has no real solutions, there are no x-intercepts.

There are no horizontal asymptotes.

Vertical asymptote: Since the denominator $x - 1 = 0$ at $x = 1$, this is a vertical asymptote.

Critical points satisfy $y'(x) = 0$, so $x(x-2) = 0$ or $x = 0$ and $x = 2$. With $y(0) = -2$, $(0, -2)$ is a maximum. (We see that $y' > 0$ for $x < 0$ and $y' < 0$ for $x > 0$, giving the maximum.) Similarly, since $y(2) = 2$, $(2, 2)$ is a minimum.

The graph is below.



9. a. For the chalone model, $P_{n+1} = f(P_n) = \dfrac{2P_n}{1+(0.05P_n)^2} = \dfrac{2P_n}{1+0.0025P_n^2}$ with $P_0 = 10$, the next three generations are

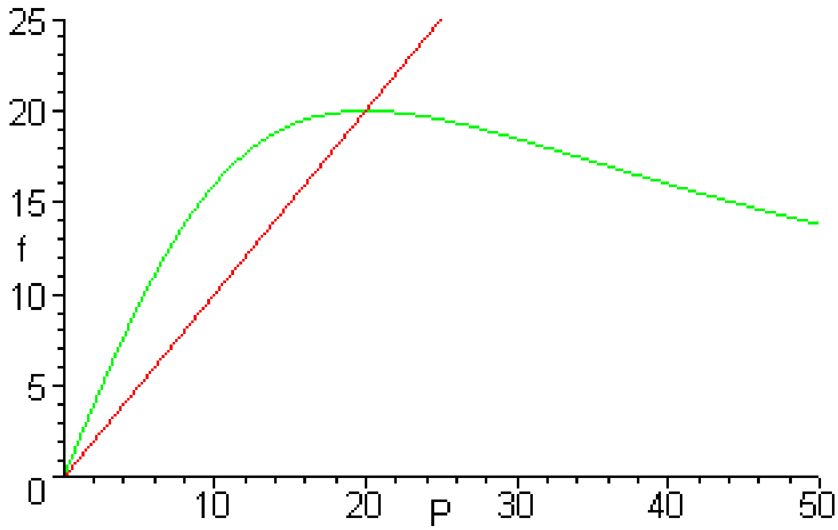
$$\begin{aligned} P_1 &= \frac{2(10)}{1+(0.05(10))^2} = 16 \\ P_2 &= \frac{2(16)}{1+(0.05(16))^2} = 19.5 \\ P_3 &= \frac{2(19.5)}{1+(0.05(19.5))^2} = 19.99 \end{aligned}$$

b. The derivative of $f(P)$ uses the quotient rule, so

$$f'(P) = \frac{2(1+0.0025P^2) - 2P(0.005P)}{(1+0.0025P^2)^2} = \frac{2-0.005P^2}{(1+0.0025P^2)^2}.$$

The maximum of $f(P)$ occurs when $f'(P) = 0$, which is when $2 - 0.005P^2 = 0$ or $P = \pm 20$. Thus, $f(20) = 20$, so the maximum occurs at $(20, 20)$. There is an intercept at $(0, 0)$. As $P \to \infty$, the exponent in the denominator is larger than the exponent in the numerator, so $f(P) \to 0$, giving the horizontal asymptote $P_{n+1} = 0$. The graph of $f(P)$ with the identity function is below.

c. At equilibrium, $P_{n+1} = P_n = P_e$, so $P_e = \frac{2P_e}{1+0.0025P_e^2}$. It follows that $P_e(1 + 0.0025P_e^2) = 2P_e$. so either $P_e = 0$ or $0.0025P_e^2 = 1$. The latter case gives $P_e = 20$. At $P_e = 0$, $f'(0) = 2 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 20$, $f'(20) = 0$, which means the solution monotonically approaches the equilibrium, so is stable.



11. a. For Hassell's model, $P_{n+1} = H(P_n) = \dfrac{5P_n}{1+0.004P_n}$. with $P_0 = 100$, the next three generations are

$$\begin{aligned} P_1 &= \frac{5(100)}{1+0.004(100)} = 357 \\ P_2 &= \frac{5(357)}{1+0.004(357)} = 735 \\ P_3 &= \frac{5(735)}{1+0.004(735)} = 933 \end{aligned}$$

b. The derivative of $H(P)$ uses the quotient rule, so

$$H'(P) = \frac{5(1+0.004P) - 5P(0.004)}{(1+0.004P)^2} = \frac{5}{(1+0.004P)^2}.$$

Since $H'(P) > 0$, the function is always increasing (no extrema). There is an intercept at $(0,0)$. The $\lim_{P\to\infty} H(P) = \frac{5}{0.004}$, giving the horizontal asymptote $P_{n+1} = 1250$. The graph of $H(P)$ with the identity function is below.

c. At equilibrium, $P_{n+1} = P_n = P_e$, so $P_e = \frac{5P_e}{1+0.004P_e}$. It follows that $P_e(1+0.004P_e) = 5P_e$. so either $P_e = 0$ or $0.004P_e^2 = 4$. The latter case gives $P_e = 1000$. At $P_e = 0$, $H'(0) = 5 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 1000$, $H'(1000) = \frac{5}{25} = \frac{1}{5} < 1$, which means the solution monotonically approaches the equilibrium, so is stable.