

Find the derivatives of the following functions:

1.  $f(x) = \frac{x^3 - \ln(x)}{1 - x^2} + \frac{2}{x^2},$

2.  $f(x) = \frac{x^2 - e^{-x}}{3x + 1} + xe^{-x},$

3.  $f(x) = \frac{\sqrt{x}}{2 + x} - \frac{1}{e^{3x}},$

4.  $f(x) = \frac{x^2 + 5}{x^2 - e^x} - \frac{xe^{2x}}{2x + 1}.$

Find the derivative and sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the  $x$  and  $y$ -intercepts and any asymptotes if they exist.

5.  $y = \frac{x^2}{x + 1},$

6.  $y = \frac{e^x}{x + 1},$

7.  $y = \frac{x^2 - 2x + 2}{x - 1},$

8.  $y = \frac{x^2}{x^2 + 1},$

9. Consider the chalone model for mitosis given by the equation

$$P_{n+1} = f(P_n) = \frac{2P_n}{1 + (bP_n)^c},$$

where  $b = 0.05$  and  $c = 2$ .

- a. Let  $P_0 = 10$ , then find  $P_1$ ,  $P_2$ , and  $P_3$ .
- b. Sketch a graph of  $f(P)$  with the identity function for  $P \geq 0$ , showing the intercepts, all extrema, and any asymptotes.
- c. Find all equilibria of the model and describe the behavior of these equilibria.

10. Repeat Exercise 10 with  $b = 0.02$  and  $c = 5$ .

11. Some entomologists use Hassell's model for studying the population of insects. Let  $P_n$  be the population of a species of beetle in week  $n$  and suppose that Hassell's model is given by

$$P_{n+1} = H(P_n) = \frac{aP_n}{1 + bP_n}.$$

Suppose that the best fit to a set of data gives  $a = 5$  and  $b = 0.004$  for this species of beetle.

- a. Let  $P_0 = 100$ , then find  $P_1$ ,  $P_2$ , and  $P_3$ .
- b. Sketch a graph of  $H(P)$  with the identity function for  $P \geq 0$ , showing the intercepts and any asymptotes.
- c. Find all equilibria of the model and describe the behavior of these equilibria.