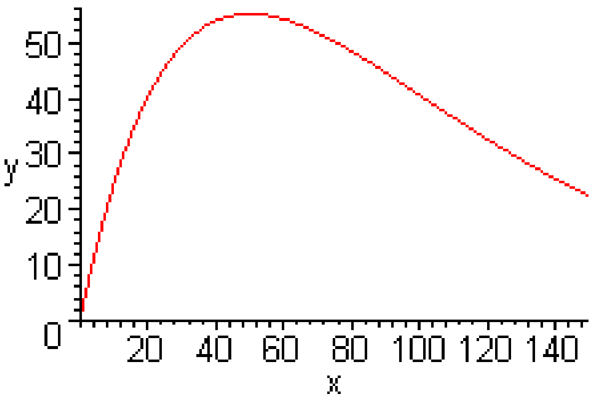
1. $f'(x) = (x^3 - 3x^2 + 7)(4x^3 - 4x + 6) + (x^4 - 2x^2 + 6x - 1)(3x^2 - 6x)$,

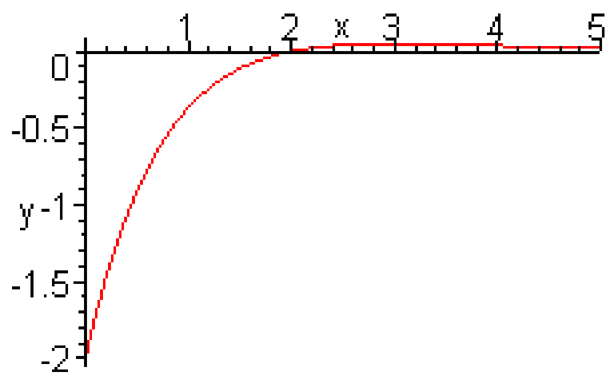
2. $f'(x) = 3(x^2 - e^{2x} + 1) + (3x + 8)(2x - 2e^{2x})$,

3. $f'(x) = (2x - x^2)e^{-x} + \frac{21}{2}x^{-1/2}$,

4. $f'(x) = x^{-3}(1 - 2\ln(x)) - 2e^{2x}(x^2 + x - 1)$.

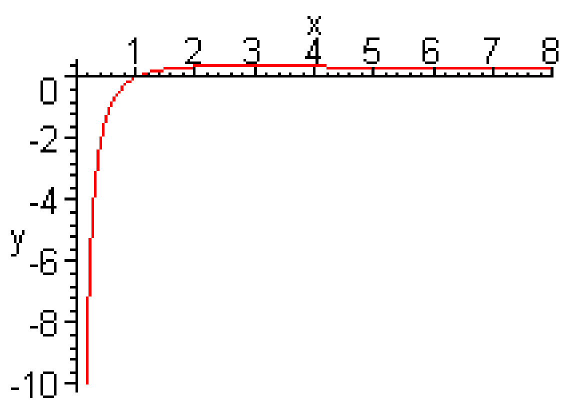
5. $y' = 3(1 - 0.02x)e^{-0.02x}$. Domain: All x. Only intercept: $(0, 0)$. Horizontal asymptote: $y = 0$. Maximum at $(50, 150e^{-1}) \simeq (50, 55.2)$. Graph is below to the left.

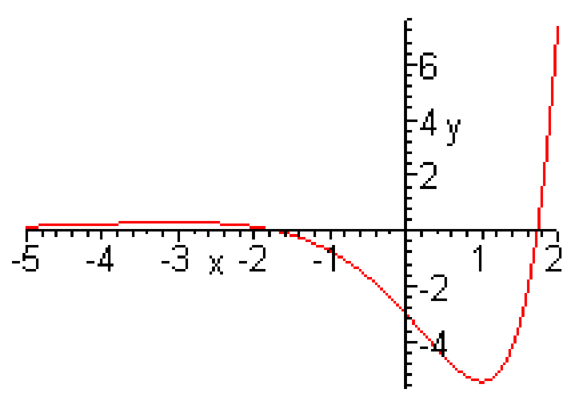




6. $y' = (3 - x)e^{-x}$. Domain: All x. y-intercept: $(0, -2)$. x-intercept: $(2, 0)$. Horizontal asymptote: $y = 0$. Maximum at $(3, e^{-3}) \simeq (3, 0.0498)$. Graph is above to the right.

7. $y' = x^{-2}(1 - \ln(x))$. Domain: $x > 0$. Only an x-intercept: $(1, 0)$. Vertical asymptote: $x = 0$. Maximum at $(e, e^{-1}) \simeq (2.72, 0.368)$. Graph is below to the left.

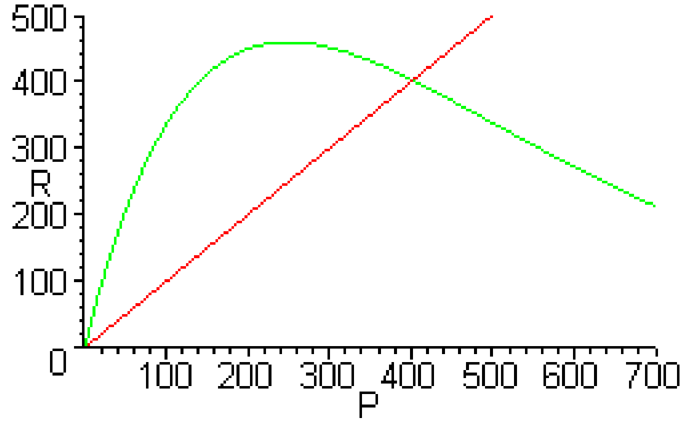




8. $y' = (x^2 + 2x - 3)e^x$. Domain: All x. y-intercept: $(0, -3)$. x-intercepts: $(\pm\sqrt{3}, 0)$. Horizontal asymptote: $y = 0$. Maximum at $(-3, 6e^{-3}) \simeq (-3, 0.2987)$ and minimum at $(1, -2e) \simeq (1, -5.437)$. Graph is above to the right.

9. a. $P_1 = 335$, $P_2 = 439$, and $P_3 = 379$.

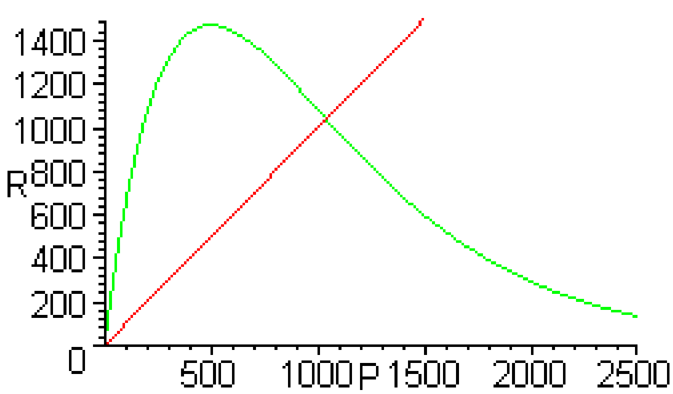
b. Below is a sketch of $R(P)$ with the identity function. Only intercept: $(0, 0)$. Maximum: $(250, 1250e^{-1}) \simeq (250, 459.8)$. Horizontal asymptote: $R = 0$.



c. The equilibria are $P_e = 0$ and $250\ln(5) \simeq 402.36$. At $P_e = 0$, $R'(0) = 5 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 250\ln(5)$, $R'(250\ln(5)) = 1 - \ln(5) \simeq -0.61$, which means the solution oscillates, but approaches the equilibrium, so is stable.

10. a. $P_1 = 655$, $P_2 = 1414$, and $P_3 = 669$.

b. Below is a sketch of $R(P)$ with the identity function. Only intercept: $(0, 0)$. Maximum: $(500, 4000e^{-1}) \simeq (500, 1471.5)$. Horizontal asymptote: $R = 0$.



c. The equilibria are $P_e = 0$ and $500\ln(8) \simeq 1039.7$. At $P_e = 0$, $R'(0) = 8 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500\ln(8)$, $R'(500\ln(8)) = 1 - \ln(8) \simeq -1.08$, which means the solution oscillates and grows away from the equilibrium, so is unstable.

11. a. $P_1 = 277$, $P_2 = 498$, and $P_3 = 486$.

b. The equilibria are $P_e = 0$ and $500\ln(8/3) \simeq 490.4$. At $P_e = 0$, $F'(0) = 3.5 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500\ln(8/3)$, $F'(500\ln(8/3)) = 1 - 1.5\ln(8/3) \simeq -0.47$, which means the solution oscillates, but approaches the equilibrium, so is stable.

c. For $h = 1$, the equilibria are $P_e = 0$ and $500\ln(2) \simeq 346.6$. At $P_e = 0$, $F'(0) = 3 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500\ln(2)$, $F'(500\ln(2)) = 1 - 2\ln(2) \simeq -0.386$, which means the solution oscillates, but approaches the equilibrium, so is stable.

For $h = 2$, the equilibria are $P_e = 0$ and $500\ln(4/3) \simeq 143.8$. At $P_e = 0$, $F'(0) = 2 > 1$, which means the solution grows monotonically away from the equilibrium, so is unstable. At $P_e = 500\ln(4/3)$, $F'(500\ln(4/3)) = 1 - 3\ln(4/3) \simeq 0.137$, which means the solution monotonically approaches the equilibrium, so is stable.

d. The fish will go extinct for any $h \geq 3$.

12. Maximum air velocity at $r = 2R/3$ with a maximum air velocity of $v(2R/3) = 4AR^3/27$.