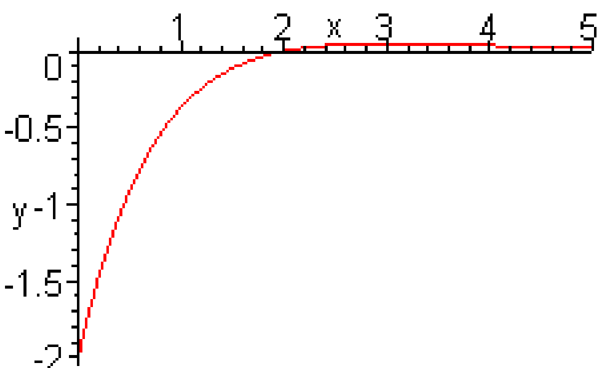
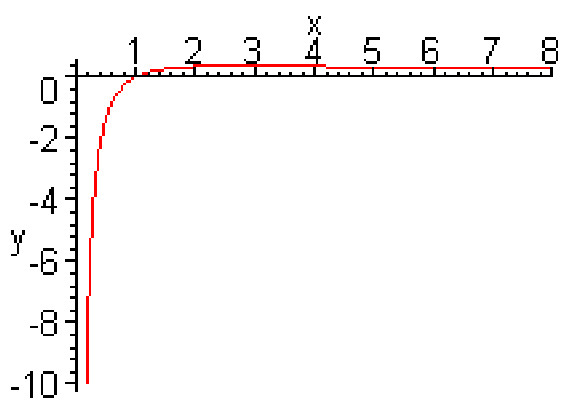
2. Consider $f(x) = (x^2 - e^{2x} + 1)(3x + 8)$, then the product rule gives $f'(x) = 3(x^2 - e^{2x} + 1) + (3x + 8)(2x - 2e^{2x})$.

4. Consider $f(x) = \frac{1}{x^2}\ln(x) - e^{2x}(x^2 - 1) = x^{-2}\ln(x) - e^{2x}(x^2 - 1)$. The product rule gives $f'(x) = x^{-2}\left(\frac{1}{x}\right) + (-2x^{-3})\ln(x) - \left(e^{2x}(2x) + 2e^{2x}(x^2 - 1)\right) = x^{-3}(1 - 2\ln(x)) - 2e^{2x}(x^2 + x - 1)$.

6. $y = (x - 2)e^{-x}$
Domain is all x.
y-intercept: $y(0) = -2$, so $(0, -2)$.
x-intercept: Since the exponential function is not zero, $y = 0$ when $x = 2$.
Horizontal asymptote: As $x \to \infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the right).
Derivative: By the product rule, $y'(x) = -(x - 2)e^{-x} + e^{-x} = (3 - x)e^{-x}$.
Critical points satisfy $y'(x) = 0$, so $3 - x = 0$ or $x = 3$. With $y(3) = e^{-3} \simeq 0.0498$, $(3, 0.0498)$ is a maximum.
The graph is below.





7. $y = \frac{1}{x}\ln(x) = x^{-1}\ln(x)$
Domain is $x > 0$.
y-intercept: None since outside the domain.
x-intercept: $y = 0$ implies that $\ln(x) = 0$ or $x = 1$.
Vertical asymptote: At the edge of the domain, so when $x = 0$.
Derivative: By the product rule, $y'(x) = x^{-1}\left(\frac{1}{x}\right) - x^{-2}\ln(x) = x^{-2}(1 - \ln(x))$.
Critical points satisfy $y'(x) = 0$, so $1 - \ln(x) = 0$ or $x = e^1 = e$. With $y(e) = e^{-1}\ln(e) = e^{-1} \simeq 0.368$, $(2.72, 0.368)$ is a maximum.
The graph is above to the right.

10. a. With Ricker's growth model, $P_{n+1} = R(P_n) = 8P_n exp(-0.002P_n)$ and $P_0 = 100$, the next three generations are

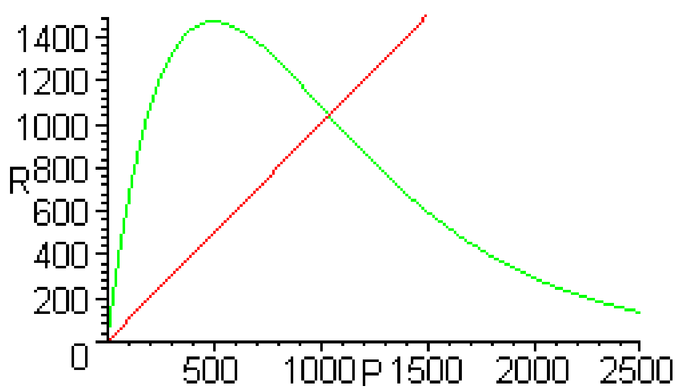
$$\begin{aligned} P_1 &= 8(100)e^{-0.002(100)} = 655 \\ P_2 &= 8(655)e^{-0.002(655)} = 1414 \\ P_3 &= 8(1414)e^{-0.002(1414)} = 669 \end{aligned}$$

b. The derivative of $R(P)$ uses the product rule, so

$$R'(P) = 8P(-0.002)e^{-0.002P} + 8e^{-0.002P} = 8(1 - .002P)e^{-0.002P}.$$

The maximum of $R(P)$ occurs when $R'(P) = 0$, which is when $1 - 0.002P = 0$ or $P = 500$. Thus, $R(500) = 4000e^{-1} = 1471.5$, so the maximum occurs at $(500, 4000e^{-1}) \simeq (500, 1471.5)$. As $P \to \infty$, the exponential dominates the polynomial part, so $R(P) \to 0$, giving the horizontal asymptote $R = 0$. The graph of the Ricker's function $R(P)$ with the identity function is below.

c. At equilibrium, $P_{n+1} = P_n = P_e$, so $P_e = 8P_e exp(-0.002P_e)$. It follows that $P_e - 8P_e exp(-0.002P_e) = P_e(1 - 8exp(-0.002P_e)) = 0$, so either $P_e = 0$ or $8exp(-0.002P_e) = 1$. The latter case gives $P_e = 500\ln(8) \simeq 1039.7$. At $P_e = 0$, $R'(0) = 8 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 500\ln(8)$, $R'(500\ln(8)) = 1 - \ln(8) \simeq -1.08$, which means the solution oscillates and grows away from the equilibrium, so is unstable.



11. a. With Ricker's growth model including fishing, $P_{n+1} = F(P_n) = 4P_n exp(-0.002P_n) - 0.5P_n$ and $P_0 = 100$, the next three generations are

$$\begin{aligned} P_1 &= 4(100)e^{-0.002(100)} - 0.5(100) = 277 \\ P_2 &= 4(277)e^{-0.002(277)} - 0.5(277) = 498 \\ P_3 &= 4(498)e^{-0.002(498)} - 0.5(498) = 486 \end{aligned}$$

b. At equilibrium, $P_{n+1} = P_n = P_e$, so $P_e = 4P_e exp(-0.002P_e) - 0.5P_e$. It follows that $1.5P_e - 4P_e exp(-0.002P_e) = P_e(1.5 - 4exp(-0.002P_e)) = 0$, so either $P_e = 0$ or $exp(0.002P_e) = \frac{4}{1.5} = \frac{8}{3}$. The latter case gives $P_e = 500\ln(8/3) \simeq 490.4$. For analyzing stability, we differentiate $F(P)$, so

$$F'(P) = 4P(-0.002e^{-0.002P}) + 4e^{-0.002P} - 0.5 = 4(1 - 0.002P)e^{-0.002P} - 0.5.$$

At $P_e = 0$, $F'(0) = 3.5 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 500\ln(8/3)$, $F'(500\ln(8/3)) = 4(1 - \ln(8/3))e^{-ln(8/3)} - 0.5 = 4(1 - \ln(8/3))\left(\frac{3}{8}\right) - 0.5 = 1 - 1.5\ln(8/3) \simeq -0.47$, which means the solution oscillates, but approaches the equilibrium, so is stable.

d. For arbitrary fishing intensity h, the equilibria satisfy $P_e = 4P_e exp(-0.002P_e) - hP_e$. It follows that $(1+h)P_e - 4P_e exp(-0.002P_e) = P_e(1 + h - 4exp(-0.002P_e)) = 0$, so either $P_e = 0$ or $exp(0.002P_e) = \frac{4}{1+h}$. The latter case gives $P_e = 500\ln\left(\frac{4}{1+h}\right)$. This second equilibrium is zero if $\frac{4}{1+h} = 1$ or $h = 3$. Thus, the fish will go extinct for any $h \geq 3$.

12. The velocity of air passing through the windpipe satisfies:

$$v(r) = Ar^2(R - r) = ARr^2 - Ar^3,$$

where A and R are constants. Differentiating $v(r)$ gives

$$v'(r) = 2ARr - 3Ar^2 = Ar(2R - 3r).$$

Thus, critical points occur at $r_c = 0$ and $2R/3$. The former is clearly a minimum, so the maximum air velocity occurs at $r_c = 2R/3$ with a maximum air velocity of

$$v(2R/3) = A\left(\frac{2R}{3}\right)^2\left(R - \frac{2R}{3}\right) = \frac{4AR^3}{27}.$$