Fall 2002 Product Rule

Find the derivatives of the following functions:

1. $f(x) = (x^3 - 3x^2 + 7)(x^4 - 2x^2 + 6x - 1)$,

2. $f(x) = (x^2 - e^{2x} + 1)(3x + 8)$,

3. $f(x) = x^2 e^{-x} + 21\sqrt{x}$,

4. $f(x) = \frac{1}{x^2} \ln(x) - e^{2x}(x^2 - 1)$.

Find the derivative and sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the x and y-intercepts and any asymptotes if they exist.

5. $y = 3xe^{-0.02x}$,

6. $y = (x - 2)e^{-x}$,

7. $y = \frac{1}{x} \ln(x)$,

8. $y = (x^2 - 3)e^{x}$,

9. $y = x^2 \ln(x)$.

10. Many biologists in fishery management use Ricker's model to study the population of fish. Let P_n be the population of fish in any year n, then Ricker's model is given by

$$P_{n+1} = R(P_n) = aP_ne^{-bP_n}.$$

Suppose that the best fit to a set of data gives $a = 5$ and $b = 0.004$ for the number of fish sampled from a particular river.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of R(P) with the identity function, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

11. Repeat Exercise 10 with $a = 8$ and $b = 0.002$

12. In fishery management, it is important to know how much fishing can be done without severely harming the population of fish. A modification of Ricker's model that includes fishing is given by the model:

$$P_{n+1} = F(P_n) = aP_n e^{-bP_n} - hP_n,$$

where $a = 4$ and $b = 0.002$ are the constants in Ricker's equation that govern the dynamics of the fish population without any fishing and h is the intensity of harvesting fish.

a. Let $h = 0.5$ and $P_0 = 100$, then find P_1, P_2, and P_3.

b. With $h = 0.5$, find all equilibria for this model and describe the behavior of these equilibria.

c. Find all equilibria for this model and describe the behavior of these equilibria when $h = 1$ and $h = 2$.

d. How intense can the fishing be before this population of fish is driven to extinction? That is, find the value of h that makes the only equilibrium be zero (or less than zero).

13. When coughing, the windpipes contract to increase the velocity of air passing through the windpipe to help clear mucus. The velocity, v, at which the air flows through the windpipe depends on the radius, r of the windpipe. If R is the resting radius of the windpipe, then the velocity of air passing through the windpipe satisfies:

$$v(r) = Ar^2(R - r),$$

where A is a constant dependent on the strength of the diaphram muscles. Find the value of r that maximizes the velocity of air and determine the velocity of the air flowing through the windpipe.