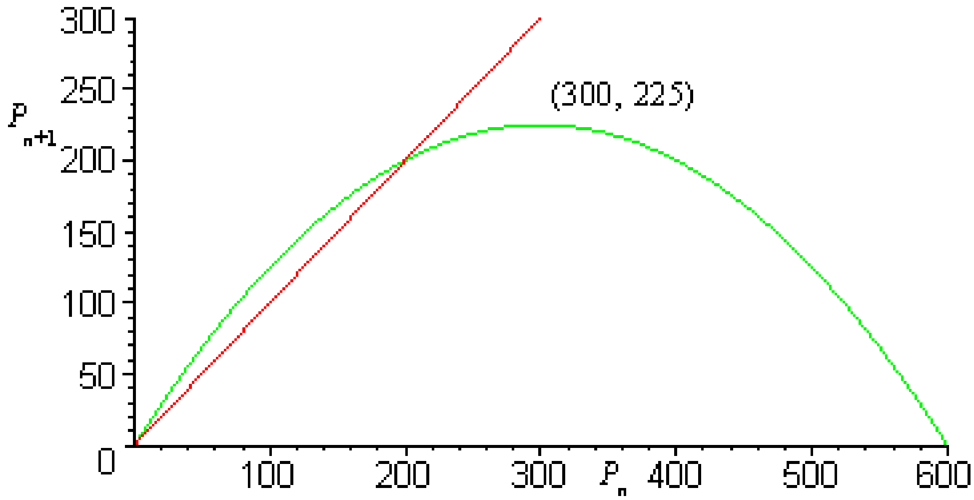
1. a. Since $P_0 = 50$,

$$\begin{aligned} P_1 &= 1.5P_0 - 0.0025P_0^2 = 1.5(50) - 0.0025(50)^2 = 68.75 \\ P_2 &= 1.5P_1 - 0.0025P_1^2 = 1.5(68.75) - 0.0025(68.75)^2 = 91.31 \\ P_3 &= 1.5P_2 - 0.0025P_2^2 = 1.5(91.31) - 0.0025(91.31)^2 = 116.12 \end{aligned}$$

b. The updating function is the quadratic function $f(p) = 1.5p - 0.0025p^2$, which has the factored form $f(p) = p(1.5 - 0.0025p)$. The p-intercepts are found by solving $f(p) = 0$, so either $p = 0$ or $1.5 - 0.0025p = 0$, which is equivalent to $p = 600$. It is easily to see that the y-intercept is 0 also. The p value of the vertex is the midpoint of these intercepts, so $p_v = 300$. Since $f(300) = 1.5(300) - 0.0025(300)^2 = 225$, it follows that the vertex occurs at $(300, 225)$.

The equilibria satisfy $P_e = 1.5P_e - 0.0025P_e^2$, so $P_e(0.5 - 0.0025P_e) = 0$. Thus, $P_e = 0$ or 200. A sketch of the graph of the updating function with the identity map, $P_{n+1} = P_n$ is show below with important points labeled.

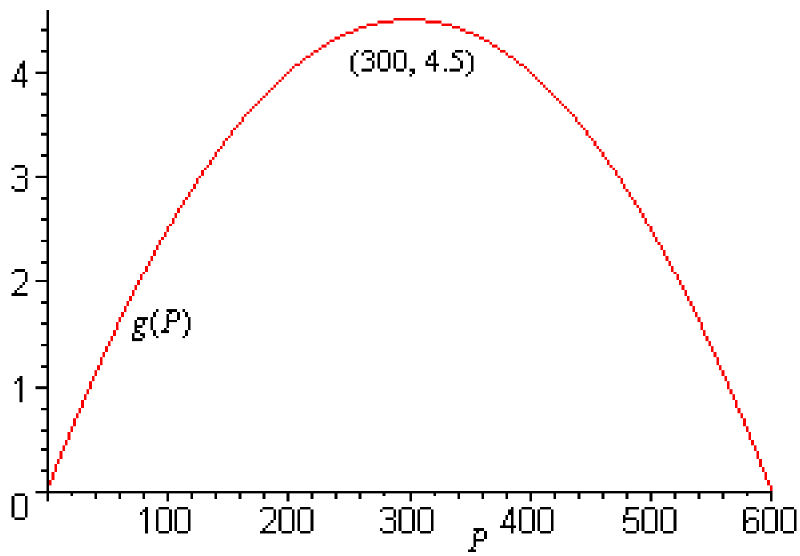


3. a. The growth rate $g(P)$ is zero when $g(P) = 0.03P(1 - P/600) = 0$. so either $P = 0$ or $(1 - P/600) = 0$, which is equivalent to $P = 600$. The p value of the vertex is the midpoint of these intercepts, so $p_v = 300$. Since $g(300) = 0.03(300)\left(1 - \frac{300}{600}\right) = 4.5$, it follows that the vertex occurs at $(300, 4.5)$. The graph is shown below.

b. Since $P_0 = 100$,

$$\begin{aligned}
P_1 &= P_0 + 0.03P_0(1 - P_0/600) = 100 + 0.03(100)\left(1 - \frac{100}{600}\right) = 102.5 \\
P_2 &= P_1 + 0.03P_1(1 - P_1/600) = 102.5 + 0.03(102.5)\left(1 - \frac{102.5}{600}\right) = 105.05 \\
P_3 &= P_2 + 0.03P_2(1 - P_2/600) = 105.05 + 0.03(105.05)\left(1 - \frac{105.05}{600}\right) = 107.65
\end{aligned}$$

The equilibria are where the growth rate is zero, so $P_e = 0$ and 600.



5. a. Since $P_0 = 500$,

$$\begin{aligned} P_1 &= 1.1P_0 - 0.0001P_0^2 - 9 = 1.1(500) - 0.0001(500)^2 - 9 = 516 \\ P_2 &= 1.1P_1 - 0.0001P_1^2 - 9 = 1.1(516) - 0.0001(516)^2 - 9 = 532 \\ P_3 &= 1.1P_2 - 0.0001P_2^2 - 9 = 1.1(532) - 0.0001(532)^2 - 9 = 548 \end{aligned}$$

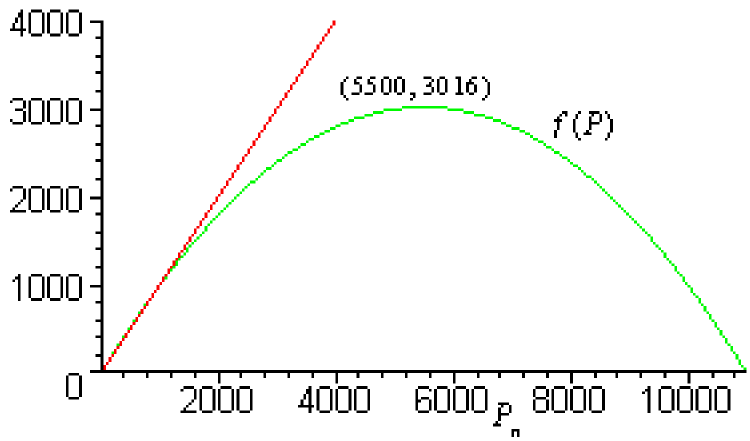
b. The updating function $f(p)$ is a quadratic, which can be written $f(p) = 1.1p - 0.0001p^2 - 9 = -0.0001(p^2 - 11000p + 90000)$. Thus, we must solve the quadratic equation $p^2 - 11000p + 90000 = 0$. The roots of this equation are found using the quadratic formula, so

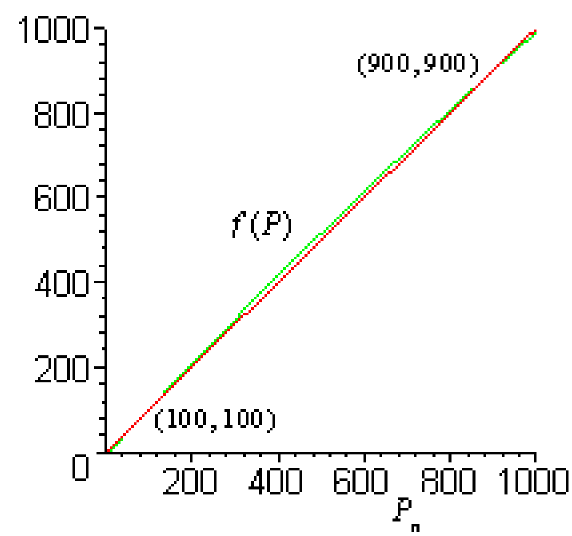
$$p = \frac{1}{2}\left(11000 \pm \sqrt{(11000)^2 - 360000}\right) \simeq 8.188, 10992.$$

The vertex is where $P_v = \frac{-b}{2a} = \frac{1.1}{0.0002} = 5500$. Since $f(5500) = 1.1(5500) - 0.0001(5500)^2 - 9 = 3016$, the vertex occurs at $(5500, 3016)$. The equilibria satisfy $P_e = 1.1P_e - 0.0001P_e^2 - 9$, so $0.0001P_e - 0.1P_e + 9 = 0$. This can be written in the factored form

$$0.0001(P_e^2 - 1000P_e + 90000) = 0.0001(P_e - 100)(P_e - 900) = 0.$$

Thus, $P_e = 100$ or 900. Two graphs are shown below with the first showing the parabola with its vertex and intercepts, while the second plot shows the equilibria where the updating function and the identity map intersect.





7. a. Since $P_0 = 500$,

$$\begin{aligned} p_1 &= \frac{5p_0}{1+0.002p_0} = \frac{5(500)}{1+0.002(500)} = 1250 \\ p_2 &= \frac{5p_1}{1+0.002p_1} = \frac{5(1250)}{1+0.002(1250)} = 1786 \\ p_3 &= \frac{5p_2}{1+0.002p_2} = \frac{5(1786)}{1+0.002(1786)} = 1953 \end{aligned}$$

b. The p-intercept satisfies $\frac{5p}{1+0.002p} = 0$, so the only point is (0.0). The horizontal asymptote is found by examining the highest powers in the numerator and denominator. Thus,

$$H(p) \simeq \frac{5p}{0.002p} = 2500,$$

since the 1 in the denominator is insignificant for very large p. The graphs of $H(p)$ with the identity map are shown below with the intersections giving the equilibria.

c. The equilibria satisfy $p_e = H(p_e)$, so

$$\begin{aligned} p_e &= \frac{5p_e}{1+0.002p_e} \\ p_e(1+0.002p_e) &= 5p_e \\ p_e(-4+0.002p_e) &= 0 \end{aligned}$$

Thus, either $p_e = 0$ or $p_e = \frac{4}{0.002} = 2000$.