

The regular solutions provide relatively complete solutions except for Problem 9.

9. The definition of the derivative is given by

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}.$$

a. For  $f(x) = x^3$ , we have

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} &= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = 3x^2. \end{aligned}$$

c. For  $f(x) = \sqrt{4-x}$ , we have

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\sqrt{4-(x+h)} - \sqrt{4-x}}{h} &= \lim_{h \rightarrow 0} \frac{(\sqrt{4-(x+h)} - \sqrt{4-x}) (\sqrt{4-(x+h)} + \sqrt{4-x})}{h (\sqrt{4-(x+h)} + \sqrt{4-x})} \\ &= \lim_{h \rightarrow 0} \frac{(4-(x+h)) - (4-x)}{h (\sqrt{4-(x+h)} + \sqrt{4-x})} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h (\sqrt{4-(x+h)} + \sqrt{4-x})} \\ &= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{4-(x+h)} + \sqrt{4-x}} = -\frac{1}{2\sqrt{4-x}} \end{aligned}$$