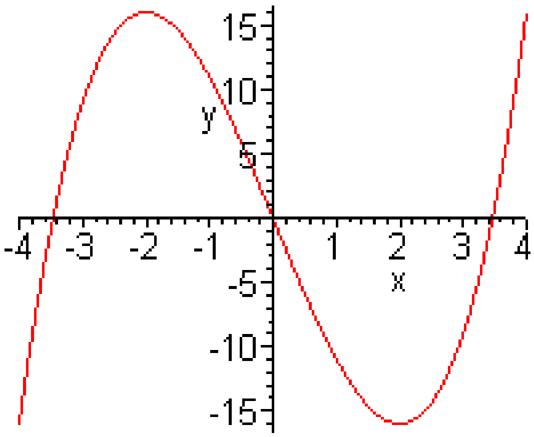
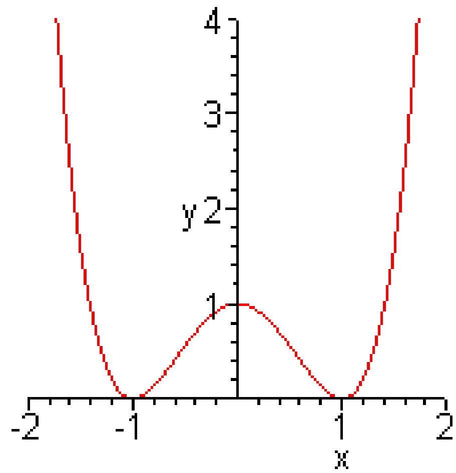
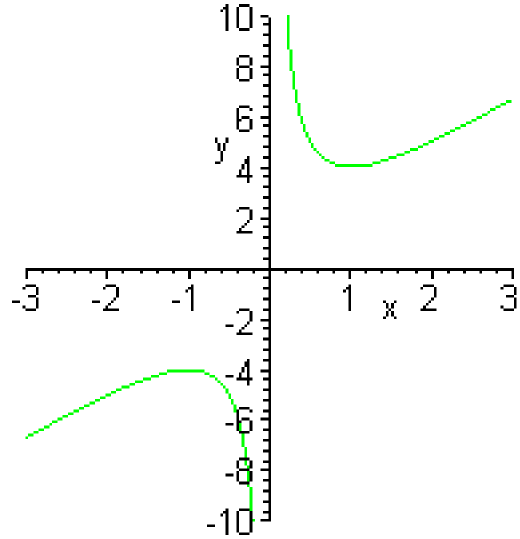
2. $y = x^3 - 12x$,
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercepts: $x^3 - 12x = x(x^2 - 12) = x(x + 2\sqrt{3})(x - 2\sqrt{3}) = 0$, so $x = 0$ and $x = \pm\sqrt{12} = \pm 2\sqrt{3}$.
No asymptotes
Derivative: $y'(x) = 3x^2 - 12$
Critical points satisfy $y'(x) = 3(x^2 - 4) = 0$, so $x_c = \pm 2$. With $y(-2) = (-2)^3 - 12(-2) = -8 + 24 = 16$ and $y(2) = -16$. Thus, $(-2, 16)$ is a maximum, and $(2, -16)$ is a minimum.
Second derivative $y''(x) = 3(2)x = 6x$.
Point of inflection ($y'' = 0$): At $x = 0$ or $(0, 0)$.





4. $y = x^4 - 2x^2 + 1$, y-intercept: $y(0) = 1$, so $(0, 1)$.
x-intercepts: $x^4 - 2x^2 + 1 = (x^2 - 1)(x^2 - 1) = (x + 1)(x - 1)(x + 1)(x - 1) = 0$, so $x = \pm 1$.
No asymptotes
Derivative: $y'(x) = 4x^3 - 4x$
Critical points satisfy $y'(x) = 4x(x^2 - 1) = 4x(x + 1)(x - 1) = 0$, so $x_c = 0, \pm 1$. With $y(0) = 1$ and $y(\pm 1) = 0$. Thus, $(0, 1)$ is a maximum, and $(\pm 1, 0)$ are minima.
Second derivative $y''(x) = 12x^2 - 4 = 4(3x^2 - 1)$.
Point of inflection ($y'' = 0$): At $x = \pm\frac{1}{\sqrt{3}}$ or $y(\pm\frac{1}{\sqrt{3}}) = \frac{1}{9} - \frac{2}{3} + 1 = \frac{4}{9}$, so $\left(\frac{1}{\sqrt{3}}, \frac{4}{9}\right)$.

6. $y = 2x + \dfrac{2}{x} = 2x + 2x^{-1}$, Vertical Asymptote at $x = 0$, so no y-intercept.
x-intercepts: $2x + \frac{2}{x} = \frac{2x^2+2}{x} = 0$, which is impossible, so there are no x-intercepts.
Derivative: $y'(x) = 2 - 2x^{-2} = \dfrac{2x^2 - 2}{x^2}$
Critical points satisfy $y'(x) = 0$ or $2(x^2 - 1) = 0$ (from the numerator above), so $x_c = \pm 1$. With $y(-1) = -4$ and $y(1) = 4$.
Second derivative $y''(x) = 4x^{-3}$.
Since $y''(-1) = -4$, the second derivative test indicates the function is concave downward at $x = -1$, so $(-1, -4)$ is a maximum. Similarly, $y''(1) = 4$, so the second derivative test shows that at $x = 1$ the point $(1, 4)$ is a minimum.
There are no points of inflection.



7. a. The discrete dynamical model for breathing is $c_{n+1} = B(c_n) = (1 - 0.15)c_n + 0.15(0.01)$. With $c_0 = 0.1$, the first two breaths satisfy

$$\begin{aligned} c_1 &= (1 - 0.15)(0.1) + (0.15)(0.01) = 0.0865 \\ c_2 &= (1 - 0.15)(0.0865) + (0.15)(0.01) = 0.0750 \end{aligned}$$

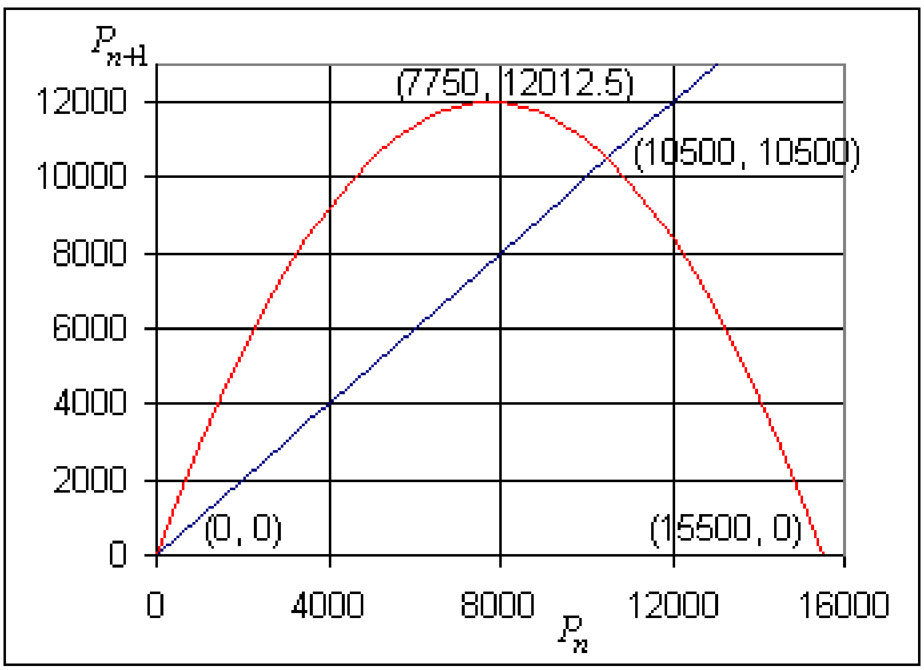
b. At equilibrium, $c_e = (1 - 0.15)c_e + (0.15)(0.01) = 0.85c_e + 0.0015$, so $0.15c_e = 0.0015$ or $c_e = 0.01$, as expected. $B'(c) = (1 - q) = 1 - 0.15 = .85$, so the derivative is less than 1, and the equilibrium is stable.

10. a. For the Logistic growth model given by $P_{n+1} = F(P_n) = 3.1P_n - 0.0002P_n^2$ with $P_0 = 1000$, the first three generations satisfy:

$$\begin{aligned} P_1 &= 3.1(1000) - 0.0002(1000)^2 = 2900 \\ P_2 &= 3.1(2900) - 0.0002(2900)^2 = 7308 \\ P_3 &= 3.1(7308) - 0.0002(7308)^2 = 11973 \end{aligned}$$

b. At equilibrium, $P_e = 3.1P_e - 0.0002P_e^2$, so $P_e(2.1 - 0.0002P_e) = 0$. Thus, either $P_e = 0$ or $P_e = \frac{2.1}{0.0002} = 10,500$.
Differentiating the updating function gives $F'(P) = 3.1 - 0.0004P$. At $P_e = 0$, $F'(0) = 3.1 > 1$, so this equilibrium is unstable with the solution monotonically moving away from the equilibrium. For $P_e = 10,500$, $F'(10500) = 3.1 - 0.0004(10500) = 3.1 - 4.2 = -1.1 < -1$. Thus. the equilibrium is again unstable, with solutions near this equilibrium oscillating and moving away from it.

c. The updating function satisfies $F(P) = P(3.1 - 0.0002P)$, which has P-intercepts at $P = 0$ and 15,500. It follows that the P value of the vertex is $P_v = 7750$ (halfway between). Since $F(7750) = 7750(3.1 - 0.0002(7750)) = 12012.5$, the vertex is located at $(7750, 12012.5)$. Below is a graph of the updating function and the identity map.



13. a. $N_0 = 4$, so $N_1 = 4 + 0.2(4)\left(1 - \frac{1}{16}(4-6)^2\right) = 4.6$ in thousands of birds. Similarly, $N_2 = 5.407$ in thousands of birds.

b. $N_e = N_e + 0.2N_e\left(1 - \frac{1}{16}(N_e - 6)^2\right)$, so $0.2N_e\left(1 - \frac{1}{16}(N_e - 6)^2\right) = 0$. Thus, $N_e = 0$ or $(N_e - 6)^2 = 16$. It follows that the equilibria are $N_e = 0$, 2, and 10.

c. The derivative is $A'(N) = \frac{3}{4} + \frac{3}{10}N - \frac{3}{80}N^2$. At $N_e = 0$, $A'(0) = \frac{3}{4}$, so this equilibrium is a stable equilibrium with solutions monotonically approaching 0. At $N_e = 2$, $A'(2) = \frac{6}{5}$, so this equilibrium is an unstable equilibrium with solutions monotonically moving away from 2. At $N_e = 10$, $A'(10) = 0$, so this equilibrium is a stable equilibrium with solutions monotonically approaching 10.

d. Biologically, these results imply that if the population is below 2 thousand, then it will go to extinction ($N_e = 0$). If the population is above 2 thousand, then the population of birds will grow to a carrying capacity of $N_e = 10$ thousand.