Sketch the curves of the functions below. List the relative maxima, relative minima, and points of inflection for each graph. Also, give the $x$ and $y$-intercepts and any asymptotes if they exist.

1. $y = 15 + 2x - x^2$,

2. $y = x^3 - 12x$,

3. $y = 2x^3 - 3x^2$,

4. $y = x^4 - 2x^2 + 1$,

5. $y = x^4 - 32x$,

6. $y = 2x + \frac{2}{x}$,

7. Our studies of a model for breathing the inert gas Argon (Ar) showed that the concentration of Ar in the lungs, $c_n$ after $n$ breaths can be given by the discrete dynamical model

$$c_{n+1} = B(c_n) = (1 - q)c_n + q\gamma,$$

where $q = 0.15$ is the fraction of air exchanged and $\gamma = 0.01$ is the atmospheric concentration of Ar.

a. Suppose that initially $c_0 = 0.1$, then find the concentrations $c_1$ and $c_2$ of Ar after the first two breaths.

b. Find the equilibrium concentration for this model. Also, determine the derivative of the updating function, $B'(c)$, and use the conditions in the lecture notes to determine the behavior of the solution near the equilibrium.

8. We studied a Malthusian growth model with emigration and showed that it satisfied the discrete dynamical model

$$P_{n+1} = G(P_n) = (1 + r)P_n - \mu,$$

where $P_n$ is the population after $n$ months, $r = 0.2$ is the rate of monthly growth for the organism, and $\mu = 1000$ is the emigration rate for this organism.

a. Suppose that initially there are 6000 individuals, so $P_0 = 6000$. Find the populations at the end of the first and second months $P_1$ and $P_2$. Repeat this calculation starting with an initial population of $P_0 = 3000$.

b. Find the equilibrium population for this model. Also, determine the derivative of the updating function, $G'(P)$, and use the conditions in the lecture notes to determine the behavior of the solution near the equilibrium.

c. What eventually happens to this population if the population begins below the equilibrium value?

9. One reasonable model for population growth was the Logistic growth model. This model is given by the discrete dynamical model

$$P_{n+1} = F(P_n) = P_n + rP_n\left(1 - \frac{P_n}{M}\right),$$

where $P_n$ is the population after $n$ generations, $r$ is the rate of growth for the organism, and $M$ is the carrying capacity of the environment for this organism.

a. Suppose that initially there are 1000 individuals, so $P_0 = 1000$. Let $r = 0.5$ and $M = 20,000$, and find the populations at the end of the first two generations $P_1$ and $P_2$.

b. Find the equilibria for this model, then use the derivative of the updating function, $F'(P)$, to determine the behavior of the solution near the equilibrium.

c. Repeat Parts a. and b. for $r = 1.1$.

10. Consider the Logistic growth model given by the discrete dynamical model

$$P_{n+1} = F(P_n) = 3.1P_n - 0.0002P_n^2,$$

where $P_n$ is the population after $n$ generations.

a. Suppose that initially there are 1000 individuals, so $P_0 = 1000$. Find the populations at the end of the first three generations $P_1$, $P_2$, and $P_3$.

b. Find the equilibria for this model, then use the derivative of the updating function, $F'(P)$, to determine the behavior of the solution near the equilibrium.

c. Sketch the updating function and the identity function ($P_{n+1} = P_n$), showing the vertex of $F(P)$, the points of intersection, and any intercepts.

11. a. An impala is migrating across a field that has been fenced with a 180 cm fence. To escape it needs to jump this fence. Assume that the impala jumps the fence with just enough vertical velocity, $v_0$ to clear it. If the height (in cm) of the impala is given by

$$h(t) = v_0 t - 490t^2,$$

then find the velocity $v(t) = h'(t)$ of the impala at any time (in sec), $t \geq 0$, before hitting the ground.

b. Find when the velocity is equal to zero in terms of $v_0$. This is the time at the maximum height. Since the impala is 180 cm in the air at this time, use the equation for the height, $h(t)$ to compute the initial velocity, $v_0$, with which the impala must launch itself to clear the fence.

c. With the initial velocity computed above, determine how long the impala is in the air, when jumping over the fence.

12. Body temperatures of animals undergo circadian rhythms. A subject's temperature is measured from 8 AM until midnight, and his body temperature, $T$ (in °C), is best approximated by the cubic polynomial

$$T(t) = 0.002(t^3 - 45t^2 + 600t + 16000),$$

where $t$ is in hours.

a. Find the rate of change in body temperature $\frac{dT}{dt}$. What is the rate of change in body temperature at noon $t = 12$?

b. Use the derivative to find when the maximum temperature of the subject occurs and when the minimum temperature of the subject occurs. What are the body temperatures at those times?

13. The San Diego Zoo discovered that because their flamingo population was too small, it would not reproduce until they borrowed some from Sea World. Scientists have discovered that certain gregarious animals require a minimum number of animals in a colony before they reproduce successfully. This is called the *Allee effect*. Consider the following model for the population of a gregarious bird species, where the population, $N_n$, is given in thousands of birds:

$$N_{n+1} = N_n + 0.2N_n\left(1 - \frac{1}{16}(N_n - 6)^2\right).$$

a. Assume that the initial population is $N_0 = 4$, then determine the population for the next two generations ($N_1$ and $N_2$).

b. Find all equilibria for this model.

c. The model above can be expanded to give

$$N_{n+1} = A(N_n) = \frac{3}{4}N_n + \frac{3}{20}N_n^2 - \frac{1}{80}N_n^3.$$

Find the derivative of $A(N)$. Evaluate the derivative $A'(N)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

d. Give a brief biological description of what your results imply about this gregarious species of bird.

14. The modeling of nerve cells often use a cubic response curve to the membrane potential $V$. Below we present a overly simple model for the membrane potential at discrete times for a nerve that can be quiescent or have repetitive spiking of action potentials. The simplified model is given by:

$$V_{n+1} = V_n + 0.07V_n(9 - (V_n - 4)^2).$$

a. Assume that the initial potential is $V_0 = 3$, then determine the membrane potential for the next three time intervals ($V_1$, $V_2$ and $V_3$).

b. Find all equilibria for this model.

c. The model above can be expanded to give

$$V_{n+1} = M(V_n) = 0.51V_n + 0.56V_n^2 - 0.07V_n^3.$$

Find the derivative of $M(V)$. Evaluate the derivative $M'(V)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

d. Give a brief biological description of what your results imply about the behavior of the nerve following different initial stimuli.