Fall 2002 Solutions Derivative of e^x and $\ln(x)$

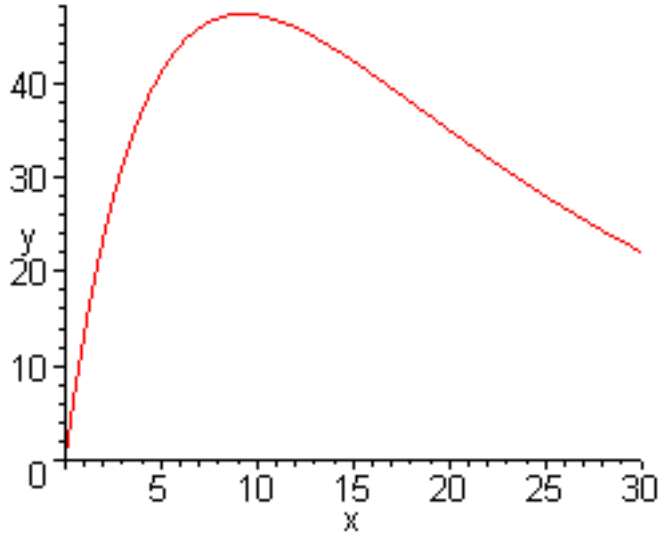
1. $f'(x) = 2x + 3e^{-x}$,

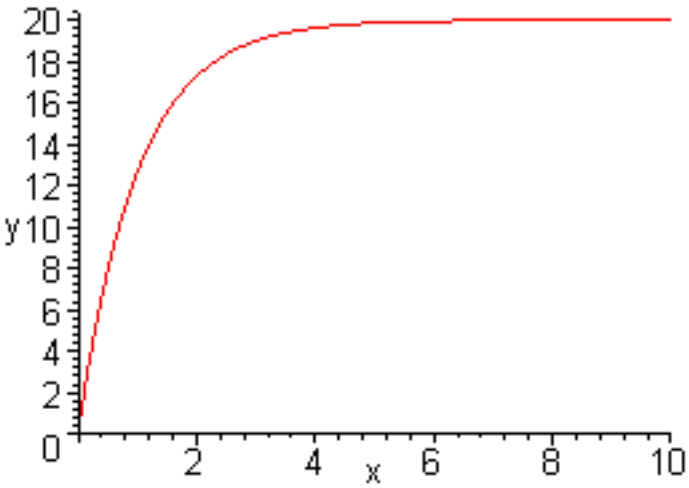
2. $f'(x) = 2 - \frac{7}{x} + 2e^{2x}$,

3. $f'(x) = -\frac{5}{x} + 2e^{-2x}$,

4. $f'(x) = -15e^{-5x} - \frac{2}{x} + x^{-2}$.

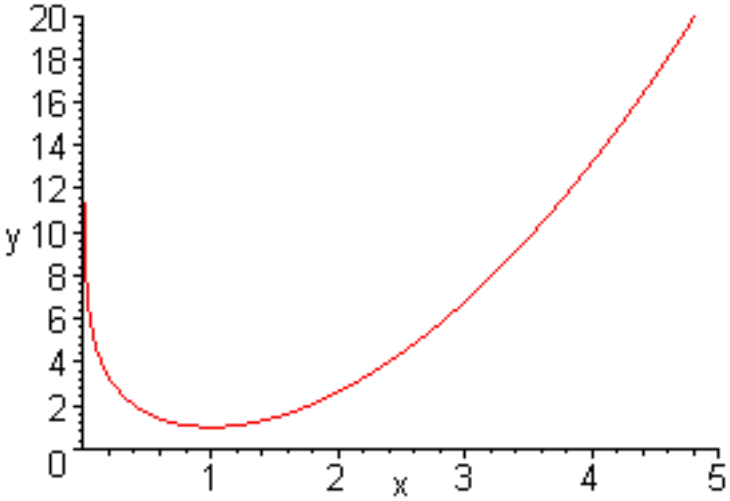
5. Domain: All x. Only intercept: $(0, 0)$. Horizontal asymptote: $y = 0$. Maximum at $(9.242, 47.25)$ ($x_c = \frac{20}{3}\ln(4)$). Graph is below to the left.

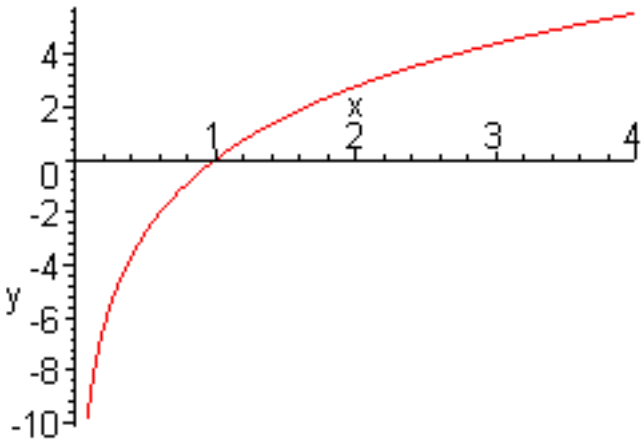




6. Domain: All x. Only intercept: $(0, 0)$. Horizontal asymptote: $y = 0$. No extrema. Graph is above to the right.

7. Domain: $x > 0$. No x or y-intercepts. Vertical asymptote: $x = 0$. Minimum at $(1, 1)$. Graph is below to the left.

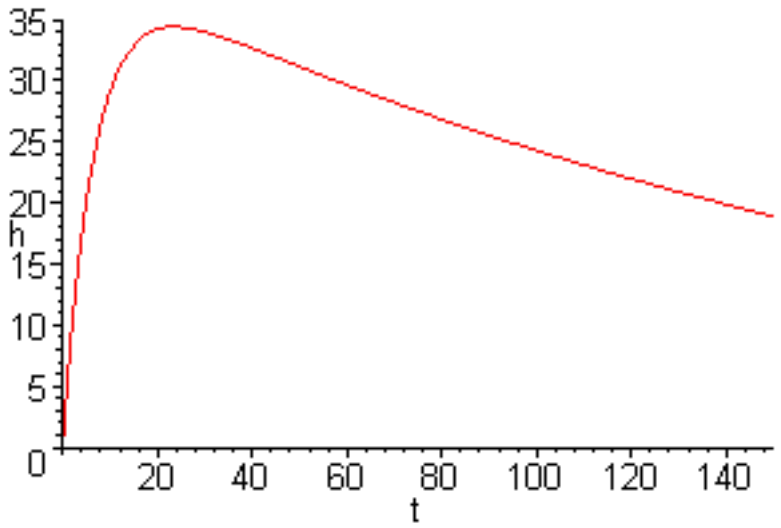




8. Domain: $x > 0$. Only an x-intercept: $(1, 0)$. Vertical asymptote: $x = 0$. No extrema. Graph is above to the right.

9. a. Maximum concentration: $h = 34.39$ ng/dl, occurring at $t = 23.46$ days.

b. Only intercept at $(0, 0)$. Horizontal asymptote: $y = 0$. The graph remains above 20 ng/dl from $t = 5.0$ to $t = 138.6$ or about 134 days. The graph of this function is below.



10. a. $a = \ln(2)/2 \simeq 0.3466$.

b. $Y'(t) = 5\ln(2)e^{t\ln(2)/2} \simeq 3.466e^{0.3466t}$

c. $Y(1) = 10\sqrt{2} \simeq 14.14$ $\quad Y'(1) = 5\ln(2)\sqrt{2} \simeq 4.901$
$Y(2) = 20$ $\quad Y'(2) = 10\ln(2) \simeq 6.931$
$Y(5) = 20\sqrt{2} \simeq 28.28$ $\quad Y'(5) = 20\ln(2)\sqrt{2} \simeq 19.61$

11. a. $a = 0.0301$

b. $P'(t) = 0.1183e^{0.0301t}$

c. $P(60) = 23.9$ million, $P'(60) = 0.720$ million/yr, $P(70) = 32.3$ million, $P'(70) = 0.972$ million/yr

d. Error for 1850 is 3.1%. Error for 1860 is 2.9%.

e. Average annual growth rate 0.82 million/yr, which is close to the average of the growth rates in Part c., which is 0.846 million/yr.

12. a. $k = \ln(2)/22 \simeq 0.0315$.

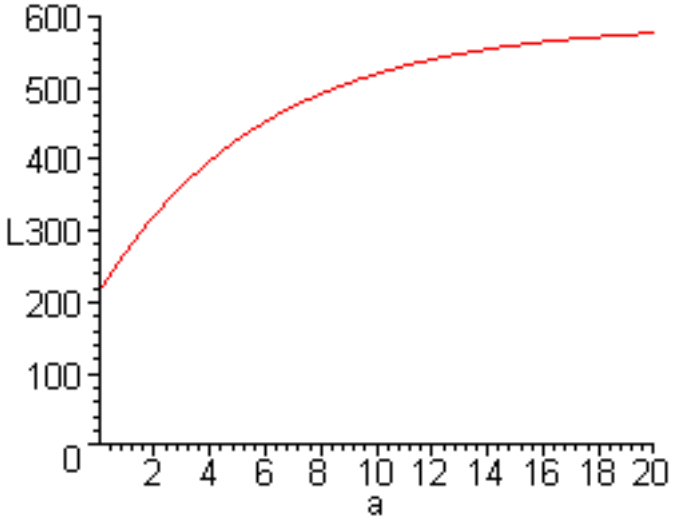
b. $R'(t) = -0.189e^{-0.0315t}$, $R'(20) = -0.101$ μg/yr, $R'(50) = -0.0391$ μg/yr, $R'(100) = -0.0081$ μg/yr.

c. 64 years

13. a. The L-intercept is $(0, 214)$. There is a horizontal asymptote at $L = 589$. The maximum length for this fish is 589 mm.

b. The fish reaches 90% of its maximum length (530.1 mm) at age $a = 11.02$ yr. The graph is shown below.

c. $L'(a) = 63.0e^{-0.168a}$, so $L'(5) = 27.2$ mm/yr.



14. a. $E'(x) = \frac{0.727}{x}$.

b. $E(10,000) = 7.47$ kJ/day, while $E'(10,000) = 7.27 \times 10^{-5}$ kJ/day/g. Biologically, the first result says that a 10 kg pronghorn fawn burns about 7.47 kJ/day in addition to the energy put into growth. The second result states that each gram of growth adds an additional 7.27×10^{-5} kJ/day of energy expended when the fawn is near 10 kg in weight.

c. The graph is shown below.