

Find the derivatives of the following functions:

1. $f(x) = x^2 - 3e^{-x} - 1,$

2. $f(x) = 2x - 7\ln(x) + e^{2x},$

3. $f(x) = 5\ln\left(\frac{1}{x}\right) - e^{-2x} + 2,$

4. $f(x) = \frac{3}{e^{5x}} + 4\ln\left(\frac{1}{\sqrt{x}}\right) - \frac{1}{x}.$

Sketch the curves of the functions below. Give the domain of each of the functions. List all maxima and minima for each graph. Also, give the x and y -intercepts and any asymptotes if they exist.

5. $y = 100(e^{-0.05x} - e^{-0.2x}),$

6. $y = 20(1 - e^{-x}),$

7. $y = x^2 - 2\ln(x),$

8. $y = 4\ln(x),$

9. Some hormones have a strong effect on mood, so finding a delivery device that delivers a hormone at a more constant level over a longer period of time is important for hormone therapy. Suppose that a drug company finds a polymer that can be implanted to deliver a hormone, $h(t)$, which is experimentally found to satisfy

$$h(t) = 40 (e^{-0.005t} - e^{-0.15t}),$$

where h is in nanograms per deciliter of blood (ng/dl) and t is in days.

- a. Find the maximum concentration of this hormone in the body and when this occurs.
- b. Determine all intercepts and asymptotes, then graph $h(t)$ for $0 \leq t \leq 150$. Use the graph to approximate how long the hormone level remains above 20 ng/dl.

10. Let $Y(t)$ be a population of yeast in a sugar solution that begins with a concentration of 10 yeast/ml.

- a. If the concentration of yeast is given by

$$Y(t) = 10e^{at},$$

then find the value of a assuming that the concentration doubles every 2 hours.

- b. Differentiate this function to find the rate of increase in the concentration of yeast per hour.
- c. Evaluate the concentration of yeast at $t = 1, 2,$ and 5 hours, and the rate of increase in the concentration of yeast per hour.

11. In an earlier section, we studied the population of the U. S. The population in 1790 was 3.93 million, and the growth rate was about 35% per decade.

a. If the population $P(t)$ is increasing exponentially, then the population at time t can be described by

$$P(t) = 3.93e^{at},$$

where P is in millions and t is in years after 1790. The population in 1800 is 5.31 million. , Determine a in the expression above.

b. Differentiate this function to find the function which represents the annual rate of growth (in millions/yr).

c. Use the expressions in Parts a. and b. to estimate the population in 1850 and 1860 and the annual growth rates at each of those dates.

d. If the actual populations in 1850 and 1860 are 23.2 and 31.4 million, respectively, then determine the percent error between this model and the actual populations.

e. Take the difference of the populations in 1850 and 1860 and divide by 10 to estimate the annual growth rate for that decade and compare that value to the values you obtained in Part c.

12. White lead, ^{210}Pb , is a radioactive element that appears in the pigment of paints and can be used to date oil paintings. This helps determine modern art forgeries. ^{210}Pb undergoes a β - decay to ^{210}Bi . Radioactive substances decay at a rate proportional to the amount of the substance available.

a. Suppose that a 1 g sample of paint contains $6 \mu\text{g}$ of ^{210}Pb . The amount of ^{210}Pb , $R(t)$ satisfies the equation,

$$R(t) = 6e^{-kt},$$

where the constant k is to be determined. If the half-life of ^{210}Pb is 22 years, then find k .

b. Find $R'(t)$, then determine the rate of change in the amount of ^{210}Pb at $t = 20, 50,$ and 100 years.

c. Suppose a fresh 1 g sample of pigment gives 60 counts per minute (cpm) (from the β decay of the ^{210}Pb), and a 1 g sample of the same pigment from a historic painting releases 8 cpm, estimate the age of the painting.

13. The cutlassfish is a valuable resource in the marine fishing industry in China. A von Bertalanffy model is fit to data for one species of this fish giving the length of the fish, $L(t)$ (in mm), as a function of the age, a (in yr). An estimate of the length of this fish is

$$L(a) = 589 - 375e^{-0.168a}.$$

a. Find the L -intercept and any asymptotes. What is the maximum possible length of this fish?

b. Determine how long it takes for this fish to reach 90% of its maximum length. Sketch a graph of the von Bertalanffy model.

c. Differentiate $L(a)$ with respect to a , then determine how fast the average fish is growing when it is 5 years old.

14. The field metabolic rate (FMR) or the total energy expenditure per day in excess of growth is calculated for pronghorn fawns using Nagy's formula

$$E(x) = 0.774 + 0.727 \ln(x),$$

where x is the mass of the fawn (in g) and $E(x)$ is the energy expenditure (in kJ/day).

a. Compute the derivative $E'(x)$.

b. Find the energy expenditure when $x = 10,000$, then compute $E'(10,000)$. Give a biological interpretation of these results.

c. Graph $E(x)$ for $x \in [5000, 20000]$.