2. The function $f(x) = x^2(x^3 - 2x + 1)^3$ is the product of two functions with the second requiring the chain rule. The derivative of $f_2(x) = (x^3 - 2x + 1)^3$ is $f_2'(x) = 3(x^3 - 2x + 1)^2(3x^2 - 2)$. By the product rule,

$$f'(x) = x^2\left[3(x^3 - 2x + 1)^2(3x^2 - 2)\right] + 2x(x^3 - 2x + 1)^3.$$

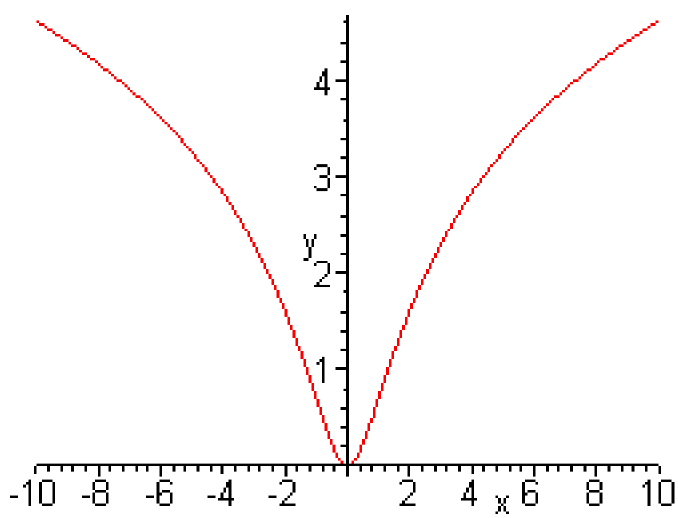
4. The function $f(x) = (x^2 - e^{-x^2})^3$ is the composition of the functions $g(u) = u^3$ and $h(x) = x^2 - e^{-x^2}$. The derivative of $h(x)$ uses the chain rule with $h'(x) = 2x - e^{-x^2}(-2x)$, since $\frac{d(e^{k(x)})}{dx} = k'(x)e^{k(x)}$. Also, $g'(u) = 3u^2$. Combining these results with $u = x^2 - e^{-x^2}$ gives

$$f'(x) = 3(x^2 - e^{-x^2})^2(2x + 2xe^{-x^2}).$$

6. For $y = \ln(x^2 + 1)$, the derivative is $y' = \dfrac{2x}{x^2 + 1}$, since by the chain rule, $\frac{d(\ln(g(x))}{dx} = \frac{g'(x)}{g(x)}$.
This is an even function.
The only intercept is $(0, 0)$.
There are no asymptotes, since the argument is strictly greater than zero.
Critical points occur when $y' = 0$, so the numerator $2x = 0$ or $x = 0$.
There is a minimum at $(0, 0)$.
The second derivative uses the quotient rule, giving

$$y'' = \frac{2(x^2 + 1) - 2x(2x)}{(x^2 + 1)^2} = \frac{2(1 - x^2)}{(x^2 + 1)^2}.$$

Thus, there are points of inflection at $(\pm 1, \ln(2)) \simeq (\pm 1, 0.693)$.
The graph is below.



8. a. For Hassell's model, $P_{n+1} = H(P_n) = \dfrac{5P_n}{(1+0.002P_n)^4}$. with $P_0 = 100$, the next three generations are

$$\begin{aligned} P_1 &= \frac{5(100)}{(1+0.002(100))^4} = 241.1 \\ P_2 &= \frac{5(241.1)}{(1+0.002(241.1))^4} = 249.8 \\ P_3 &= \frac{5(249.8)}{(1+0.002(249.8))^4} = 248.7 \end{aligned}$$

b. The derivative of $H(P)$ uses the quotient rule and chain rule, so

$$H'(P) = \frac{5(1+0.002P)^4 - 20P(1+0.002P)^3(0.002)}{(1+0.002P)^8} = \frac{5(1+0.002P - 4(0.002)P)}{(1+0.002P)^5} = \frac{5(1-0.006P)}{(1+0.002P)^5}.$$

There is a critical point when the numerator of the derivative is zero, so $1 - 0.006P_c = 0$ or $P_c = \frac{500}{3}$. It follows that there is a maximum at $(500/3, 625(3/4)^3) \simeq (166.7, 263.7)$. There is an intercept at $(0,0)$. The $\lim_{P\to\infty} H(P) = 0$, giving the horizontal asymptote $P_{n+1} = 0$. The graph of $H(P)$ with the identity function is below.

c. At equilibrium, $P_{n+1} = P_n = P_e$, so $P_e = \dfrac{5P_e}{(1+0.002P_e)^4}$. It follows that $P_e(1 + 0.002P_e)^4 = 5P_e$. so either $P_e = 0$ or $(1+0.002P_e)^4 = 5$. The latter case gives $1+0.002P_e = 5^{1/4}$, so $P_e = 500(5^{1/4} - 1) \simeq 247.7$. At $P_e = 0$, $H'(0) = 5 > 1$, so this equilibrium is unstable with solutions monotonically growing away from $P_e = 0$. At $P_e = 500(5^(1/4)-1)$, $H'(500(5^{1/4}-1)) = (4\cdot 5^{3/4} - 15)/5 \simeq -0.325$, which means the solution oscillates, but approaches the equilibrium, so is stable.