This Lecture Activity has you actively work with the lecture notes presented in class and available on my website. This activity is due by **Tues. Nov 23 by noon**. The problems below require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

1. (6pts) In lecture we examined a transcendental equation with a small term in the algebraic equation. This problem examines another transcendental equation with a small nonlinear perturbation:

$$x^2 - x - 6 = \varepsilon \cos(x), \qquad \text{with} \quad \varepsilon \ll 1.$$

This equation has two solutions, which cannot be solved algebraically. Use the techniques from lecture with

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \mathcal{O}\left(\varepsilon^3\right),$$

to find x_0, x_1, and x_2 for any ε. Then find approximate solutions for $\varepsilon = 0.1$ and $\varepsilon = 0.01$, showing the approximate values for both solutions with a two term ($\mathcal{O}(\varepsilon)$) expansion and a three term ($\mathcal{O}(\varepsilon^2)$) expansion. Compare these values to the "actual" ones found using a nonlinear solver, like *fsolve* from either Maple or MatLab. (Slides 10-11)

2. (5pts) a. Consider the nonlinear IVP given by:

$$\frac{dy}{dt} + y = \varepsilon y^3, \qquad \text{with} \quad y(0) = 1.$$

This is a Bernoulli's equation, which is readily solved from techniques shown in Math 337 and in this class. Find the exact solution for any ε, then find the power series expansion in ε up through $\mathcal{O}(\varepsilon^2)$ (a three term expansion).

b. (5pts) Use the techniques from lecture, assuming a solution in the form:

$$y(t) = y_0(t) + \varepsilon y_1(t) + \varepsilon^2 y_2(t) + \mathcal{O}\left(\varepsilon^3\right),$$

with the initial conditions:

$$y_0(0) = 1, \qquad y_1(0) = y_2(0) = \cdots = 0.$$

Find y_0, y_1, and y_2, that approximate the solution to the nonlinear ODE above. Compare these iterative solutions to the power series for the exact solution. (Slides 16-19)