This Lecture Activity has you actively work with the lecture notes presented in class and available on my website. This activity is due by **Mon. Sep 20 by noon**. There are 3 problems that require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

1. (5pts) Consider the example:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -0.5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Find the general solution to this problem. Classify the type of equilibrium and create a phase portrait using either MatLab (pplane8) or Maple. Also, determine a fundamental solution. (Slides Fundamental 3–5)

2. (4pts) In lecture we used our definitions to prove equivalence of $\|\cdot\|_1$ and $\|\cdot\|_2$. Use definitions and possibly the Cauchy-Schwartz inequality to prove the equivalence of $\|\cdot\|_\infty$ and $\|\cdot\|_2$, *i.e.*, show that

$$C\|\mathbf{x}\|_\infty \leq \|\mathbf{x}\|_2 \leq D\|\mathbf{x}\|_\infty,$$

for some constants C and D. (Slides Fundamental 10–12)

3. (7pts) a. Consider the matrices:

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Let $C = A + B$. Find $\|C\|_1$ and $\|C\|_\infty$. (Slide Fundamental 15)

b. Show that A and B commute. Using the Property of Matrix Exponential Products, find $e^{Ct} = e^{(A+B)t}$. (Slide Fundamental 19)