This Lecture Activity has you actively work with the lecture notes presented in class and available on my website. This activity is due by **Mon. Sep 13 by noon**. There are 3 problems that require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

1. (6pts) Consider the data on a collection of various sized mammals given below.

Animal	Weight	Metabolism	Animal	Weight	Metabolism
Mouse	0.021	3.6	Dog	23.6	872
Rat	0.282	28.1	Chimpanzee	38	1090
Guinea pig	0.41	35.1	Sheep	46.8	1330
Rabbit	3.57	154	Woman	57.2	1368
Cat	3	152	Heifer	482	7754

The weight, w, is in kg and the metabolism, M, is in kcal. Use an allometric or power law model to find Kleiber's Law:

$$M = Aw^r,$$

which relates the weight of an animal to its metabolism. Find the best linear fit to the logarithms of the data to obtain the best A and r. Create a graph with both the data and best linear fit, using the logarithmic scales for w and M. Create a second graph of the data and the allometric model using normal scales for w and M. Give a short discussion for why you obtain the value of r that you find, using your knowledge of how animals use their energy based on their size. (Slides Linear 37–38)

2. (5pts) The following is a spruce budworm model based on work of Ludwig, Jones, and Hollings[1]:

$$\frac{dB}{dt} = r_B B\left(1 - \frac{B}{K_B}\right) - \frac{\beta B^2}{\alpha^2 + B^2}.$$

This ODE has 4 parameters with r_B being the intrinsic growth rate, K_B being the carrying capacity depending on fir foliage, β representing saturation of the predator, and α switching factor of avian predators in a Holling's Type III predation. Scale the population and time by:

$$p = sB \qquad \text{and} \qquad \tau = qt,$$

to obtain the simpler scaled ODE:

$$\frac{dp}{d\tau} = Rp\left(1 - \frac{p}{Q}\right) - \frac{p^2}{1 + p^2},$$

which only has two parameters, Q and R. Give the scaling factors s and q and the expressions for Q and R in terms of the original parameters. Show your work for how you reduced this system. (Slides Linear 39–42)

[1] D. Ludwig, D. D. Jones, C. S. Holling (1978), "Qualitative Analysis of Insect Outbreak Systems: The Spruce Budworm and Forest," *J. Animal Ecology*, **47**,pp 315-332.

3. (5pts) Continuing Problem 2: Let $Q = 10$ and $R = 0.5$. Find all equilibria and determine the stability of all equilibria. This may require numerical/computer methods to find the equilibria, so explain how you obtained your equilibria and their stability. Draw a 1D Phase Portrait for this model. (Stability Review of Math 337: https://jmahaffy.sdsu.edu/courses/f15/math337/beamer/dfield1-04.pdf Slides 21-28)