This supplement to Exam 1 covers the Numerical Methods techniques, specifically Euler's method and Improved Euler's method, which really should be done on computers. This will be due by **Mon. Oct 18 by noon**. There are 2 problems that require written answers, including some tables of numbers and a graph, which are entered into **Gradescope**. You are to fill in the Tables below with your numerical answers, as I do not need to see all the intermediate calculations. Work should be shown for solving the first ODE. MatLab programs are provided to allow you to obtain highly accurate answers to the "actual" solution for the second problem. You will simply need to run the provided program, so **no programming** is required for this "actual" solution.

1. (15pts) Consider the initial value problem:

$$\frac{dy}{dt} = \frac{y}{t} + 2t, \qquad y(1) = 5.$$

a. Write your solution to this initial value problem, stating the type of ODE and showing the steps required to solve the problem.

b. Use Euler's method to simulate the solution for $t \in [1, 4]$ with a stepsizes of $h = 0.2$, 0.1, and 0.05. Also, compute the percent error between the Euler approximation and the actual solution at $t = 2$, 3, and 4. Use **6** significant figures for your Euler approximations and **3** significant figures for your percent errors.

t_n	Actual	Euler $h = 0.2$	Error %	Euler $h = 0.1$	Error %	Euler $h = 0.05$	Error %
2							
3							
4							

c. Use Improved Euler's method to simulate the solution for $t \in [1, 4]$ with stepsizes of $h = 0.2$, 0.1, and 0.05. Also, compute the percent error between the Improved Euler approximation and the actual solution at $t = 2$, 3, and 4. Use **6** significant figures for your Improved Euler approximations and **3** significant figures for your percent errors.

t_n	Actual	Im Euler $h = 0.2$	Error %	Im Euler $h = 0.1$	Error %	Im Euler $h = 0.05$	Error %
2							
3							
4							

d. Write a brief discussion of how your results demonstrate the global truncation error of the Euler's and Improved Euler's methods.

2. (15pts) Exam 1 had a problem, which examined the stability of the process of *induction*, which is used by bacterial cells to stimulate certain genes for catabolic functions. We could find the equilibria and determine their stability, but the governing ODE could not be solved. This problem examines the numerical solution to an ODE for induction. Let x be the concentration of a protein controlled by induction satisfying the ODE:

$$\frac{dx}{dt} = \frac{32.5\, x^2}{30 + x^2} - 2.5\, x, \qquad x(0) = 3.1,$$

where t is in μsec. As we did in the previous problem, the time varying solution is generated numerically with *Euler's* and *Improved Euler's* methods.

a. Use the supplied MatLab program to produce the "Actual solution" for $t = 1$, 2, 3, 4, and 5. Then use Euler's method to simulate the solution for $t \in [0, 5]$ with a stepsizes of $h = 0.2$, 0.1, and 0.05. Also, compute the percent error between the Euler approximation and the actual solution at $t = 1$, 2, 3, 4, and 5. Use **5** significant figures for your Euler approximations and **3** significant figures for your percent errors.

	'Actual'	Euler $h = 0.2$	Error %	Euler $h = 0.1$	Error %	Euler $h = 0.05$	Error %
t_n							
1							
2							
3							
4							
5							

b. Use Improved Euler's method to simulate the solution for $t \in [0, 5]$ with stepsizes of $h = 0.2$, 0.1, and 0.05. Also, compute the percent error between the Improved Euler approximation and the actual solution at $t = 1$, 2, 3, 4, and 5. Use **5** significant figures for your Improved Euler approximations and **3** significant figures for your percent errors.

	'Actual'	Im Euler $h = 0.2$	Error %	Im Euler $h = 0.1$	Error %	Im Euler $h = 0.05$	Error %
t_n							
1							
2							
3							
4							
5							

c. Use the Improved Euler's method with $h = 0.1$ to simulate this model for $t \in [0, 6]$ with both of the initial conditions, $x(0) = 3.1$ and $x(0) = 2.9$. Create a graph showing these approximate solutions. Returning to the behavior observed in your Exam 1 Problem 2 (with possibly different equilibria), write a brief discussion of what these two simulations show.