

From the lecture Slide 21, we have:

$$\phi(t_n + h) = \phi(t_n) + \phi'(t_n)h + \frac{1}{2}\phi''(\bar{t}_n)h^2,$$

and Euler's formula gives:

$$y_{n+1} = y_n + hf(t_n, y_n).$$

If we define  $E_n = \phi(t_n) - y_n$ , then using  $\phi' = f(t, \phi)$ ,

$$\phi(t_n + h) - y_{n+1} = [\phi(t_n) - y_n] + h[f(t_n, \phi(t_n)) - f(t_n, y_n)] + \frac{1}{2}\phi''(\bar{t}_n)h^2,$$

or

$$E_{n+1} = E_n + h[f(t_n, \phi(t_n)) - f(t_n, y_n)] + \frac{1}{2}\phi''(\bar{t}_n)h^2.$$