Project 1 – Due October 17, 2017

Be sure to follow all the directions detailed on the Cover document.

1. **Radius of the Earth Problem:** a. From a point in Mission Valley, the top of Cowles Mountain and the top of Mount Cuyumaca line up. The height above sea level of this point in Mission Valley is 40 ft, the height of Cowles Mountain is 1591 ft, and the height of Mount Cuyumaca is 6512 ft. The distance between this point in Mission Valley and Cowles Mountain is 9.20 miles and between the point in Mission Valley and Mount Cuyumaca is 34.85 miles. From these observations calculate the radius of the Earth.

The solution should include a figure, showing the key distances and variables in the problem. All equations required for the solution need to be included. Decide on an appropriate numerical technique to solve this problem, giving a reason for selecting this technique and providing a MatLab code that solves the problem. Do not forget to say how appropriate initial guesses are provided for the numerical routine.
Hint: Center a coordinate system at the center of the Earth with the point in Mission Valley having coordinates (Radius + height, 0). The polar coordinates of the mountain tops will then be given by (Radius + height, Angle) where the Angle can be determined by the definition of radian measure (Angle = Dist/Radius). Now write the coordinates of these two mountain tops in Cartesian coordinates. Since the line Mission Valley to Cowles and the line Mission Valley to Cuyumaca are the same, they have the same slopes. Compute and equate these slopes to derive the equation in the variable R. Now solve for R.

b. Suppose the the distances to and heights of Cowles Mountain and Mount Cuyumaca might each have an error of as much as $\pm 0.1\%$ (*i.e.,* the height of Cowles Mountain is 1591 ± 1.6 ft). (You can assume that the original height in Mission Valley is correct.) With this information calculate the range of distances for the radius of the Earth. That is, find the minimum and maximum radius for the Earth if these potential errors are taken into account. Which combination of the **4** possible errors gives the minimum radius, and which combination gives the maximum radius?

2. **Logistic Growth for U. S. Population:** The logistic growth model is described by an equation of the form:

$$P(t) = \frac{P_L}{1 - ce^{-kt}},$$

where P_L, c, and k are constants, and $P(t)$ is the population at time t. The constant P_L is known as the carrying capacity and represents the limiting population for the logistic growth model. The table below gives the population of the U. S. during the twentieth century (with the population given in millions).

Year	Census	Year	Census
1900	76.212	1960	179.323
1910	92.228	1970	203.302
1920	106.022	1980	226.546
1930	122.775	1990	248.710
1940	132.165	2000	281.422
1950	150.697	2010	308.746

a. Use the populations in 1900, 1930, and 1960 with $t = 0$ at 1900 to find the constants P_L, c, and k. Describe how one finds these parameters. Decide on an appropriate numerical technique to solve this problem, giving a reason for selecting this technique and providing a MatLab code that solves the problem. Do not forget to say how appropriate initial guesses are provided for the numerical routine.

b. Use this model to predict the population in 1980 and 2000. Taking the actual census data as the best predicted value, find the percent error between the model and the census data. (Use percent error as the signed relative error or $100\times$(model - census)/census.) What is the predicted limiting population for the U.S. according to this logistic growth model?

c. Create a graph showing the data and the model. Describe how well the model fits the data. Give a brief discussion on how a better fit to the entire set of data might be performed. (Do NOT actually find this "best fit;" however, this will be discussed later in the course.)