>

Below we see how to enter a function in Maple.

> $f := x \rightarrow \cos(\cos(x));$

$$f := x \rightarrow \cos(\cos(x)) \quad (1)$$

The statement below integrates the function from 0 to Pi/2. The indefinite integral (which cannot be done in this case) would simply have an x after the comma.

> $int\left(f(x), x = 0 .. \frac{\text{Pi}}{2}\right);$

$$\frac{1}{2}\,\pi\,\text{BesselJ}(0, 1) \quad (2)$$

The $evalf(\%)$ command evaluates the previous expression $(\%)$ **in** floating point arithmetic, **then** stores the value **in** F.

> $F := evalf(\%);$

$$F := 1.201969716 \quad (3)$$

The *series* command allows you to produce a Taylor series. The x=0 says that this series is in x and is centered at 0. The 10 at the end gives the number of terms in the series. Equivalently, you can use the *taylor* command. The *evalf* command obtains decimal coefficients.

> $series(f(x), x = 0, 10);$

$$\cos(1) + \frac{1}{2}\sin(1)\,x^2 + \left(-\frac{1}{8}\cos(1) - \frac{1}{24}\sin(1)\right)x^4 + \left(\frac{1}{48}\cos(1) - \frac{7}{360}\sin(1)\right)x^6 + \left(\frac{1}{960}\cos(1) + \frac{209}{40320}\sin(1)\right)x^8 + \mathrm{O}(x^{10}) \quad (4)$$

> $taylor(f(x), x = 0, 10);$

$$\cos(1) + \frac{1}{2}\sin(1)\,x^2 + \left(-\frac{1}{8}\cos(1) - \frac{1}{24}\sin(1)\right)x^4 + \left(\frac{1}{48}\cos(1) - \frac{7}{360}\sin(1)\right)x^6 + \left(\frac{1}{960}\cos(1) + \frac{209}{40320}\sin(1)\right)x^8 + \mathrm{O}(x^{10}) \quad (5)$$

> $evalf(\%);$

$$0.5403023059 + 0.4207354924\,x^2 - 0.1025990793\,x^4 - 0.00510563777\,x^6 + 0.004924606465\,x^8 + \mathrm{O}(x^{10}) \quad (6)$$

We can readily convert this into a polynomial by either copying the terms we want into a new function or using the *convert* function. The *convert* command produces an expression P, to which we use the *unapply* command to create a function of x.

> $P := convert(\%, polynom);\ P8 := unapply(P, x);$

$$P := 0.5403023059 + 0.4207354924\,x^2 - 0.1025990793\,x^4 - 0.00510563777\,x^6 + 0.004924606465\,x^8$$

$$P8 := x \rightarrow 0.5403023059 + 0.4207354924\,x^2 - 0.1025990793\,x^4 - 0.00510563777\,x^6 + 0.004924606465\,x^8 \quad (7)$$

> $P8(1);$

$$0.8582576877 \quad (8)$$

We can chain these commands together to obtain the quadratic we are seeking with ; separating the commands.

> $series(f(x), x=0, 3); evalf(\%);$
> $P := convert(\%, polynom); P2 := unapply(P, x);$

$$\cos(1) + \frac{1}{2}\sin(1)\,x^2 + O(x^4)$$

$$0.5403023059 + 0.4207354924\,x^2 + O(x^4)$$

$$P := 0.5403023059 + 0.4207354924\,x^2$$

$$P2 := x \rightarrow 0.5403023059 + 0.4207354924\,x^2 \quad \textbf{(9)}$$

We integrate this second order polynomial over the interval 0 to Pi/2 and find the % error compared to the actual value.

> $IP := int\left(P2(x), x=0..\frac{Pi}{2}\right); \frac{100\cdot(IP-F)}{F};$

$$IP := 1.392264923$$

$$15.83194688 \quad \textbf{(10)}$$

We can individually find the derivatives to compute the Taylor coefficients with the *diff* command. The *x*\$2 means taking the 2nd derivative.

> $diff(f(x), x);$

$$\sin(\cos(x))\sin(x) \quad \textbf{(11)}$$

> $diff(f(x), x\$2);$

$$-\cos(\cos(x))\sin(x)^2 + \sin(\cos(x))\cos(x) \quad \textbf{(12)}$$

Next we want to obtain the Taylor's series about *x*0 = Pi/4 and show how to produce LaTeX code for the result.

> $x0 := \frac{Pi}{4};$

$$x0 := \frac{1}{4}\pi \quad \textbf{(13)}$$

> $series(f(x), x=x0, 10);$

$$\cos\left(\frac{1}{2}\sqrt{2}\right) + \frac{1}{2}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\left(x-\frac{1}{4}\pi\right) + \left(-\frac{1}{4}\cos\left(\frac{1}{2}\sqrt{2}\right) + \frac{1}{4}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^2 + \left(-\frac{1}{4}\cos\left(\frac{1}{2}\sqrt{2}\right) - \frac{1}{8}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^3 + \left(\frac{1}{32}\cos\left(\frac{1}{2}\sqrt{2}\right) - \frac{1}{12}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^4 + \left(\frac{1}{12}\cos\left(\frac{1}{2}\sqrt{2}\right) - \frac{1}{192}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^5 + \left(\frac{1}{128}\cos\left(\frac{1}{2}\sqrt{2}\right) + \frac{139}{5760}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^6 + \left(-\frac{79}{5760}\cos\left(\frac{1}{2}\sqrt{2}\right) + \frac{607}{80640}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^7 + \left(-\frac{715}{129024}\cos\left(\frac{1}{2}\sqrt{2}\right) - \frac{11}{2688}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x-\frac{1}{4}\pi\right)^8 + \left(\frac{19}{13440}\cos\left(\frac{1}{2}\sqrt{2}\right) \quad \textbf{(14)}$$

$$- \frac{24167}{11612160} \sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\left(x - \frac{1}{4}\pi\right)^9 + O\left(\left(x - \frac{1}{4}\pi\right)^{10}\right)$$

> *evalf*(%);

$$0.7602445972 + 0.4593626847\,(x - 0.7853981635) + 0.0396201931\,(x - 0.7853981635)^2 - 0.3049018205\,(x - 0.7853981635)^3 - 0.05280280379\,(x - 0.7853981635)^4 + 0.05856868846\,(x - 0.7853981635)^5 + 0.02811004049\,(x - 0.7853981635)^6 - 0.003511461127\,(x - 0.7853981635)^7 - 0.007972639835\,(x - 0.7853981635)^8 - 0.000837282702\,(x - 0.7853981635)^9 + O\left((x - 0.7853981635)^{10}\right) \quad \textbf{(15)}$$

> *latex*(%);

```
( 0.7602445972+ 0.4593626847\, \left( x- 0.7853981635 \right) +
 0.0396201931\, \left( x- 0.7853981635 \right) ^{2}-
0.3049018205\,
 \left( x- 0.7853981635 \right) ^{3}- 0.05280280379\, \left( x-
 0.7853981635 \right) ^{4}+ 0.05856868846\, \left( x-
0.7853981635
 \right) ^{5}+ 0.02811004049\, \left( x- 0.7853981635 \right) ^
{6}-
 0.003511461127\, \left( x- 0.7853981635 \right) ^{7}-
0.007972639835
\, \left( x- 0.7853981635 \right) ^{8}- 0.000837282702\, \left(
x-
 0.7853981635 \right) ^{9}+O \left(  \left( x- 0.7853981635
\right) ^{
10} \right) )
```

For our problem in the notes we want the second order Taylor polynomial, so as before we write:

> *series*(*f*(*x*), *x*=*x0*, 3); *evalf*(%);
> *T* := *convert*(%, *polynom*); *T2* := *unapply*(*T*, *x*);

$$\cos\left(\frac{1}{2}\sqrt{2}\right) + \frac{1}{2}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\left(x - \frac{1}{4}\pi\right) + \left(-\frac{1}{4}\cos\left(\frac{1}{2}\sqrt{2}\right) + \frac{1}{4}\sin\left(\frac{1}{2}\sqrt{2}\right)\sqrt{2}\right)\left(x - \frac{1}{4}\pi\right)^2 + O\left(\left(x - \frac{1}{4}\pi\right)^3\right)$$

$$0.7602445972 + 0.4593626847\,(x - 0.7853981635) + 0.0396201931\,(x - 0.7853981635)^2 + O\left((x - 0.7853981635)^3\right)$$

$$T := 0.3994619883 + 0.4593626847\,x + 0.0396201931\,(x - 0.7853981635)^2$$

$$T2 := x \rightarrow 0.3994619883 + 0.4593626847\,x + 0.0396201931\,(x - 0.7853981635)^2 \quad \textbf{(16)}$$

We now compare the error by integrating P2 over the first half of interval + integrating T2 over the second half of the interval as compared to the actual value.

> $I1 := int\left(P2(x), x = 0..\frac{Pi}{4}\right); I2 := int\left(T2(x), x = \frac{Pi}{4}..\frac{Pi}{2}\right); I3 := I1 + I2; \frac{100\cdot(I3 - F)}{F};$

$$I1 := 0.4922974444$$

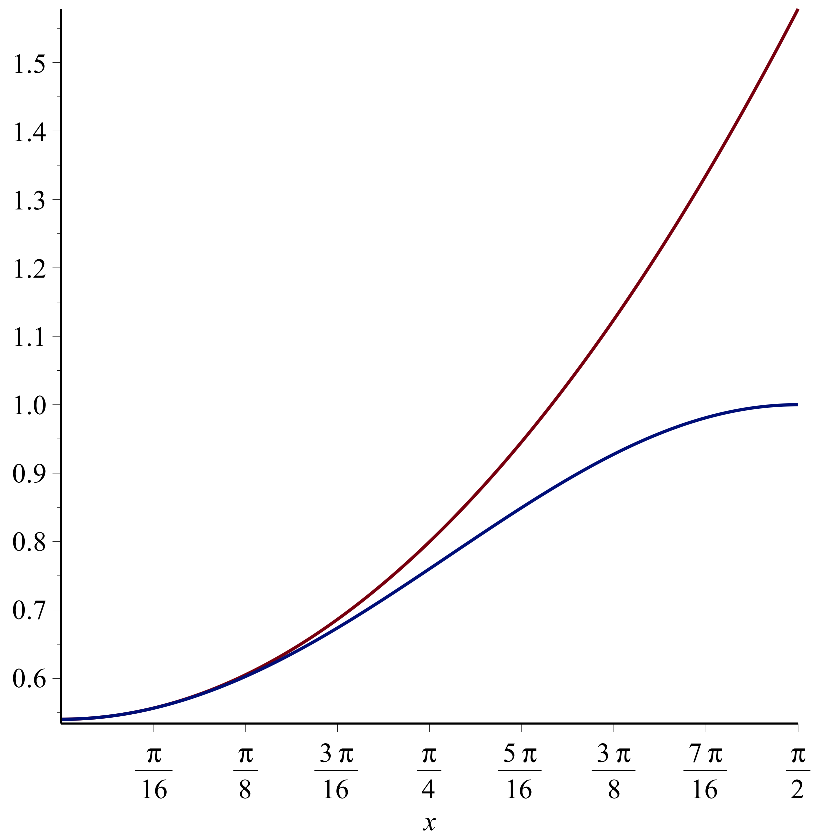
$$I2 := 0.7451720151$$

$$I3 := 1.237469460$$

$$2.953464095 \quad \textbf{(17)}$$

Let us create some graphic displays of our regions.

> $plot\left(\{f(x), P2(x)\}, x = 0..\frac{\text{Pi}}{2}\right);$

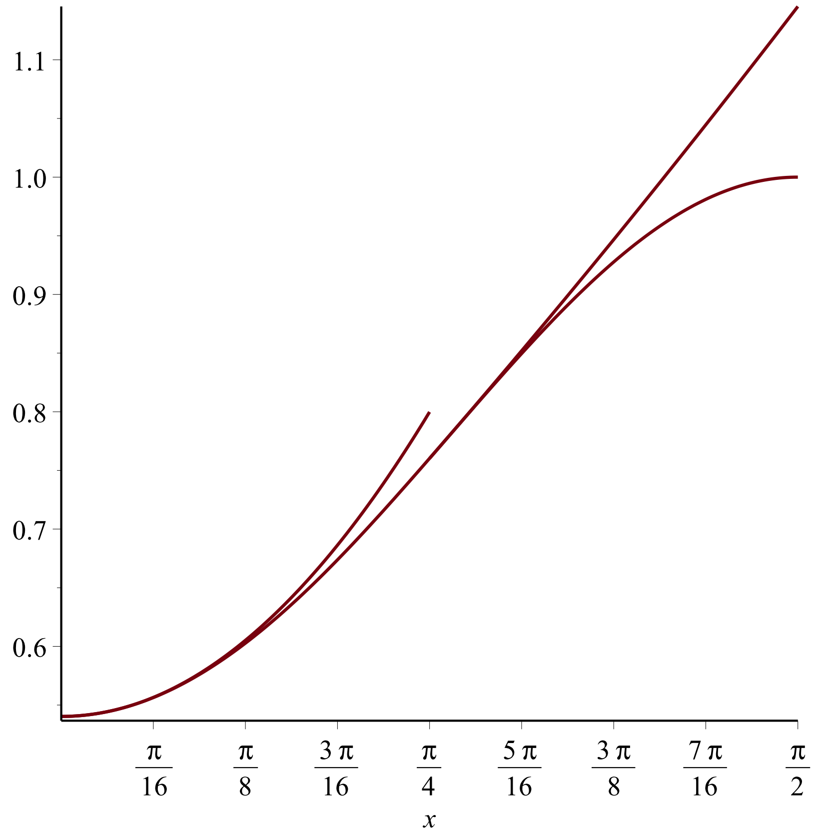


With different domains we need overlapping plots and a special package.

> $with(plots):$

> $Pt1 := plot\left(f(x), x = 0..\frac{\text{Pi}}{2}\right) : \; Pt2 := plot\left(P2(x), x = 0..\frac{\text{Pi}}{4}\right) : Pt3 := plot\left(T2(x), x = \frac{\text{Pi}}{4}..\frac{\text{Pi}}{2}\right):$

> $display(\{Pt1, Pt2, Pt3\});$



1.1
1.0
0.9
0.8
0.7
0.6
π/16
π/8
3π/16
π/4
5π/16
3π/8
7π/16
π/2
x

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