1. For the following 2^{nd} order nonhomogeneous differential equations, find the general solution. You may use any technique that works for the problem.

a. $y'' - y = 4t^2 + 6e^{-t}$,

b. $y'' - 2y' + 2y = 4\cos(t)$,

c. $y'' + 9y = 18\tan(3t)$,

d. $y'' + 4y = 8\csc(2t)$,

e. $y'' + 4y' + 4y = \dfrac{e^{-2t}}{t^2}$,

f. $t^2y'' + 7ty' + 5y = 6t$

2. Consider the following differential equation:

$$(1-t)y'' + ty' - y = 2(t-1)^2e^{-t}.$$

Verify that $y_1(t) = t$ and $y_2(t) = e^t$ solve the homogeneous problem. Use this information to find the general solution to the nonhomogeneous problem above.

3. For each of the following nonhomogeneous differential equations give the form of the particular solution that you would guess in using the **method of undetermined coefficients**. (**DO NOT** solve these equations.)

a. $y'' - 9y = 2te^{-3t}\sin(t) + 6t^2e^{3t}$,

b. $y'' + 2y' + 2y = 4t^2e^{-t} + 3te^{-t}\cos(t)$.

4. Solve the following initial value problems using the method of Laplace transforms. (Thus, you must show the problem in the transform space and the solution in t.)

a. $y'' - 4y = 16\,e^{-2t}, \qquad y(0) = 1, \quad y'(0) = 2$

b. $y'' - y' - 12y = t^2\delta(t-3), \qquad y(0) = 2, \quad y'(0) = -6$

c. $y'' + 2y' + 5y = \begin{cases} 5, & 0 \le t < 4 \\ 0, & t \ge 4 \end{cases} \qquad y(0) = 0, \quad y'(0) = 4$

d. $y'' + 4y = \cos(t), \qquad y(0) = 1, \quad y'(0) = 0$

e. $y'' + 2y' - 3y = \begin{cases} t^2, & 0 \le t < 3 \\ 0, & t \ge 3 \end{cases} \qquad y(0) = 0, \quad y'(0) = 0$

5. a. Show that, if $\mathcal{L}\{f\} = F(s)$, then

$$\mathcal{L}\{t\, f(t)\} = -\frac{dF(s)}{ds}.$$

b. From the result above, derive the formula for $\mathcal{L}\{t\ \sin(2t)\}$.

c. Use this result to solve the initial value problem

$$y'' + 4y = 2\cos(2t), \qquad y(0) = -1, \quad y'(0) = 4.$$

6. A bridge can be considered to be a harmonic oscillator. When someone is walking across the bridge, their steps impart an impulsive force. Below we examine two cases of impulsive force applied to the harmonic oscillator.

a. Solve the initial value problem:

$$\frac{d^2y}{dt^2} + y = \sum_{j=0}^{\infty} \delta(t - j\pi), \qquad y(0) = 0 \qquad y'(0) = 0.$$

b. Solve this initial value problem:

$$\frac{d^2y}{dt^2} + y = \sum_{j=0}^{\infty} \delta(t - 2j\pi), \qquad y(0) = 0 \qquad y'(0) = 0.$$

c. Use the information above to explain why soldiers are instructed to break cadence when marching across a bridge. (A historical note: In the 17^{th} century, a number of British soldiers died when they marched across a bridge and set up resonance so that the bridge collapsed.)