

```
> de1a := diff(x1(t), t) = -0.5*x1(t) + 2*x2(t);
de1b := diff(x2(t), t) = -x2(t);
```

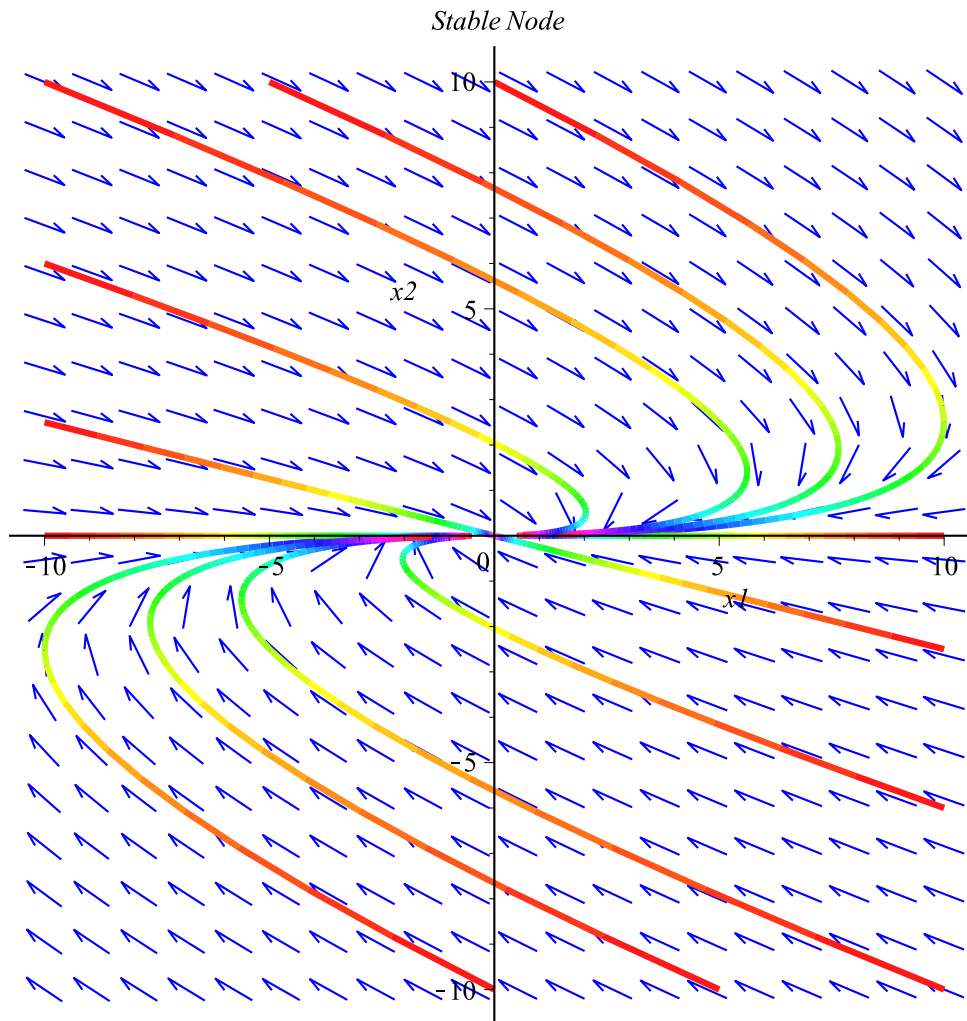
$$de1a := \frac{d}{dt} x1(t) = -0.5 x1(t) + 2 x2(t)$$

$$de1b := \frac{d}{dt} x2(t) = -x2(t)$$

(1)

```
> with(DEtools) :
```

```
> DEplot( {de1a, de1b}, [x1(t), x2(t)], t=0..6, [[x1(0) = 0, x2(0) = 10], [x1(0) = 10, x2(0) = 0], [x1(0) = 0, x2(0) = -10], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = 2.5], [x1(0) = 10, x2(0) = -2.5], [x1(0) = -5, x2(0) = 10], [x1(0) = -10, x2(0) = 10], [x1(0) = 10, x2(0) = -6], [x1(0) = 10, x2(0) = -10], [x1(0) = -10, x2(0) = 6], [x1(0) = 5, x2(0) = -10]], x1 = -10..10, x2 = -10..10, title='Stable Node', color = blue, linecolor = t);
```



```
> de2a := diff(x1(t), t) = x2(t);
```

$$de2b := \text{diff}(x2(t), t) = -3 \cdot x1(t) + 4 \cdot x2(t);$$

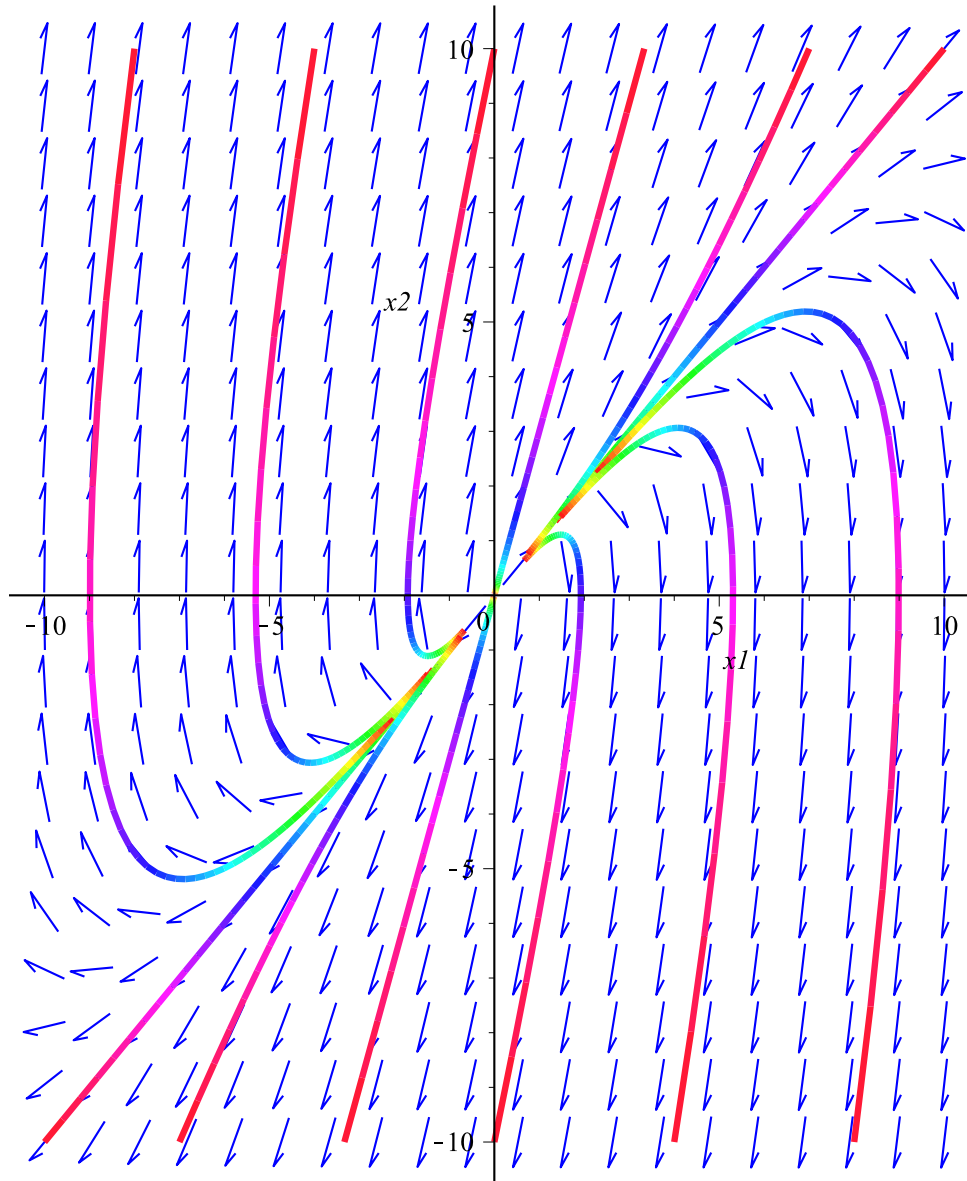
$$de2a := \frac{d}{dt} x1(t) = x2(t)$$

$$de2b := \frac{d}{dt} x2(t) = -3 x1(t) + 4 x2(t)$$

**(2)**

>  $DEplot(\{de2a, de2b\}, [x1(t), x2(t)], t=-2..0, \left[ [x1(0) = 10, x2(0) = 10], [x1(0) = -10, x2(0) = -10], [x1(0) = -8, x2(0) = 10], [x1(0) = 8, x2(0) = -10], \left[ x1(0) = \frac{10}{3}, x2(0) = 10 \right], \left[ x1(0) = -\frac{10}{3}, x2(0) = -10 \right], [x1(0) = 0, x2(0) = 10], [x1(0) = 0, x2(0) = -10], [x1(0) = 7, x2(0) = 10], [x1(0) = -4, x2(0) = 10], [x1(0) = -7, x2(0) = -10], [x1(0) = 4, x2(0) = -10] \right], x1 = -10..10, x2 = -10..10, title = 'Unstable Node', color = blue, linecolor = t);$

Unstable Node



>  $de3a := \text{diff}(x1(t), t) = x1(t) + 3 \cdot x2(t);$   
 $de3b := \text{diff}(x2(t), t) = x1(t) - x2(t);$

$$de3a := \frac{d}{dt} x1(t) = x1(t) + 3 x2(t)$$

$$de3b := \frac{d}{dt} x2(t) = x1(t) - x2(t)$$

(3)

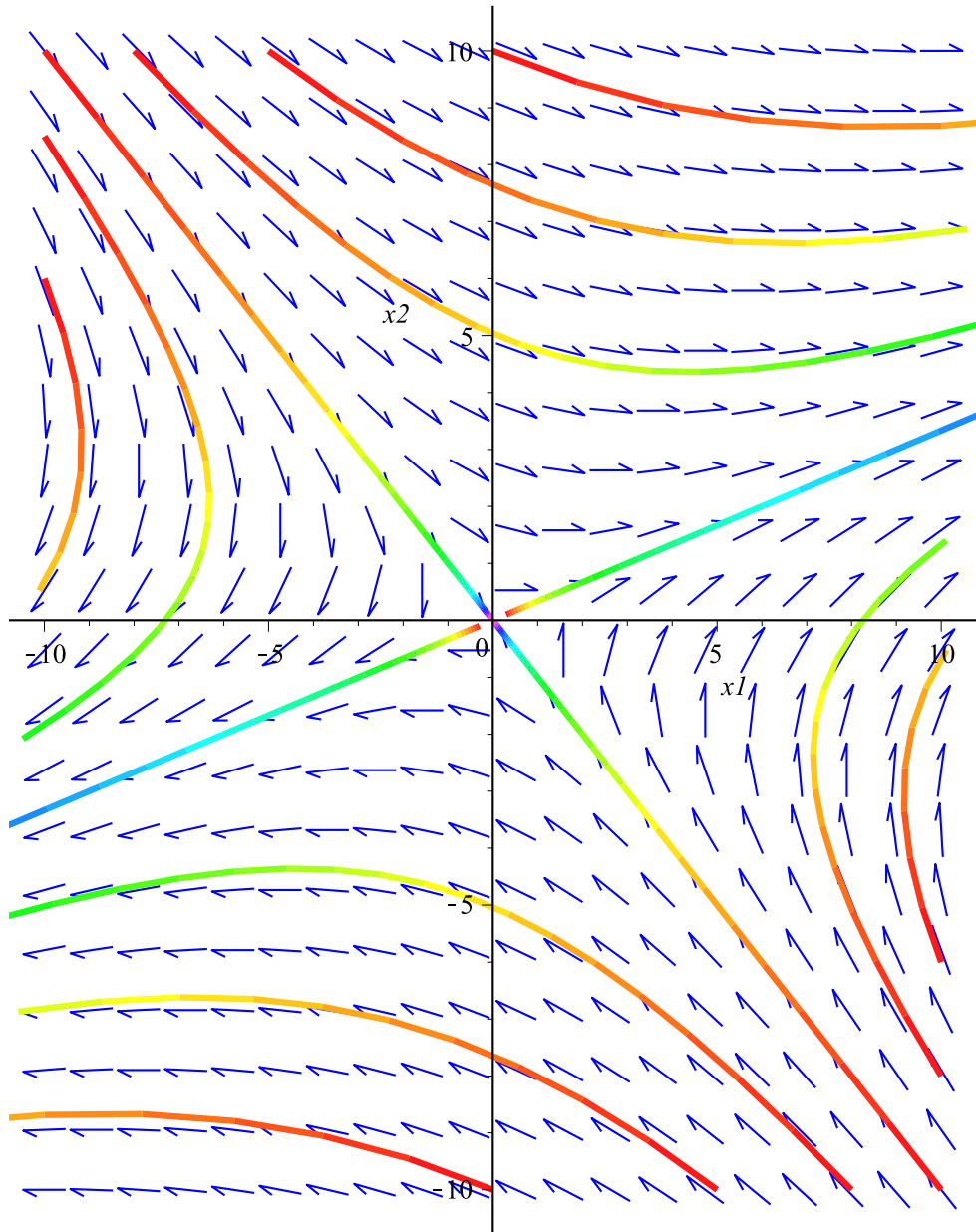
>  $DEplot(\{de3a, de3b\}, [x1(t), x2(t)], t=0..3, [[x1(0) = -10, x2(0) = 10], [x1(0) = 10, x2(0) =$

```

-10], [x1(0) = -0.3, x2(0) = -0.1], [x1(0) = 0.3, x2(0) = 0.1], [x1(0) = -10, x2(0) = 8.5],
[x1(0) = -10, x2(0) = 6], [x1(0) = 10, x2(0) = -8.], [x1(0) = 10, x2(0) = -6], [x1(0) =
-8, x2(0) = 10], [x1(0) = -5, x2(0) = 10], [x1(0) = 8, x2(0) = -10], [x1(0) = 5, x2(0) =
-10], [x1(0) = 0, x2(0) = -10], [x1(0) = 0, x2(0) = 10], x1 = -10..10, x2 = -10..10, title =
'Saddle Node', color = blue, linecolor = t);

```

Saddle Node



> with(plots) :

>  $de4a := \text{diff}(x1(t), t) = -2 \cdot x1(t) + 4 \cdot x2(t);$   
 $de4b := \text{diff}(x2(t), t) = x1(t) - 2 \cdot x2(t);$

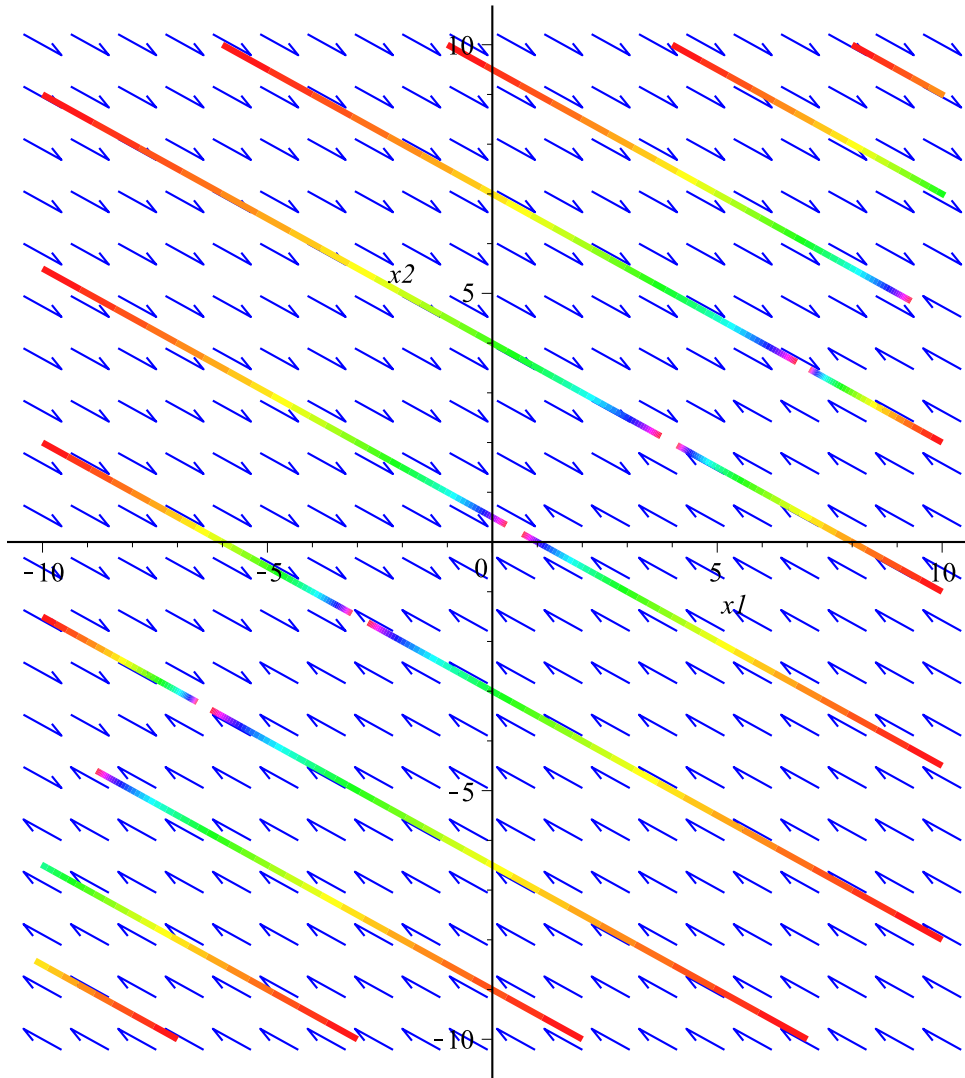
$$de4a := \frac{d}{dt} x1(t) = -2 x1(t) + 4 x2(t)$$

$$de4b := \frac{d}{dt} x2(t) = x1(t) - 2 x2(t)$$

**(4)**

>  $DEplot(\{de4a, de4b\}, [x1(t), x2(t)], t=0..1, [[x1(0)=-10, x2(0)=-1.5], [x1(0)=-10, x2(0)=2], [x1(0)=-10, x2(0)=5.5], [x1(0)=-10, x2(0)=9], [x1(0)=-6, x2(0)=10], [x1(0)=-1, x2(0)=10], [x1(0)=4, x2(0)=10], [x1(0)=8, x2(0)=10.], [x1(0)=10, x2(0)=2], [x1(0)=10, x2(0)=-1], [x1(0)=10, x2(0)=-4.5], [x1(0)=10, x2(0)=-8], [x1(0)=7, x2(0)=-10], [x1(0)=2, x2(0)=-10], [x1(0)=-3, x2(0)=-10], [x1(0)=-7, x2(0)=-10]], x1=-10..10, x2=-10..10, title='Zero Eigenvalue - stable', color=blue, linecolor=t);$

Zero Eigenvalue – stable

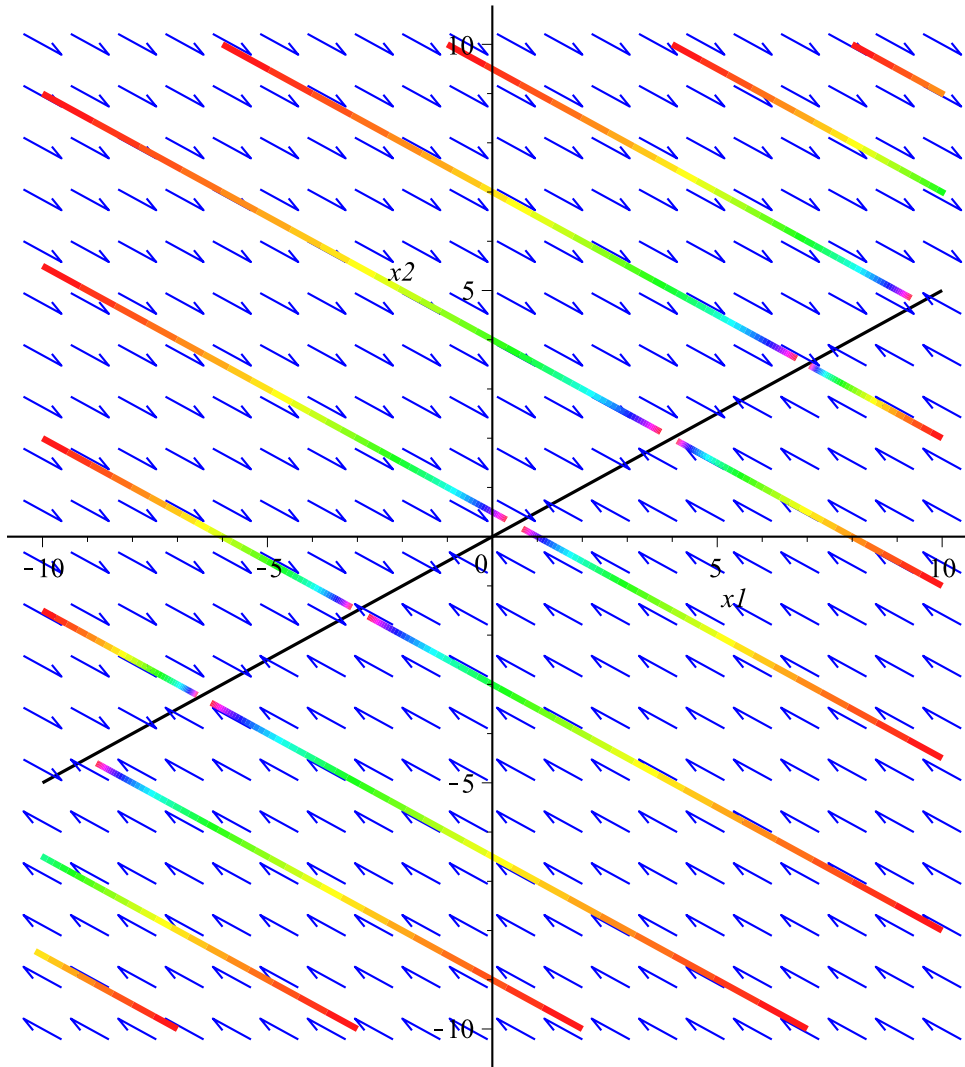


```
> P1 := DEplot( {de4a, de4b}, [x1(t), x2(t)], t=0..1, [[x1(0)=-10, x2(0)=-1.5], [x1(0)=-10, x2(0)=2], [x1(0)=-10, x2(0)=5.5], [x1(0)=-10, x2(0)=9], [x1(0)=-6, x2(0)=10], [x1(0)=-1, x2(0)=10], [x1(0)=4, x2(0)=10], [x1(0)=8, x2(0)=10.], [x1(0)=10, x2(0)=2], [x1(0)=10, x2(0)=-1], [x1(0)=10, x2(0)=-4.5], [x1(0)=10, x2(0)=-8], [x1(0)=7, x2(0)=-10], [x1(0)=2, x2(0)=-10], [x1(0)=-3, x2(0)=-10], [x1(0)=-7, x2(0)=-10]], x1=-10..10, x2=-10..10, title='Zero Eigenvalue - stable', color=blue, linecolor=t) :
```

```
> P2 := plot(0.5·x1, x1=-10..10, x2=-10..10, color=black) :
```

```
> display( {P1, P2} );
```

Zero Eigenvalue – stable



```
> de5a := diff(x1(t), t) = 3*x1(t) - 2*x2(t);
de5b := diff(x2(t), t) = 4*x1(t) - x2(t);
```

$$de5a := \frac{d}{dt} x1(t) = 3 x1(t) - 2 x2(t)$$

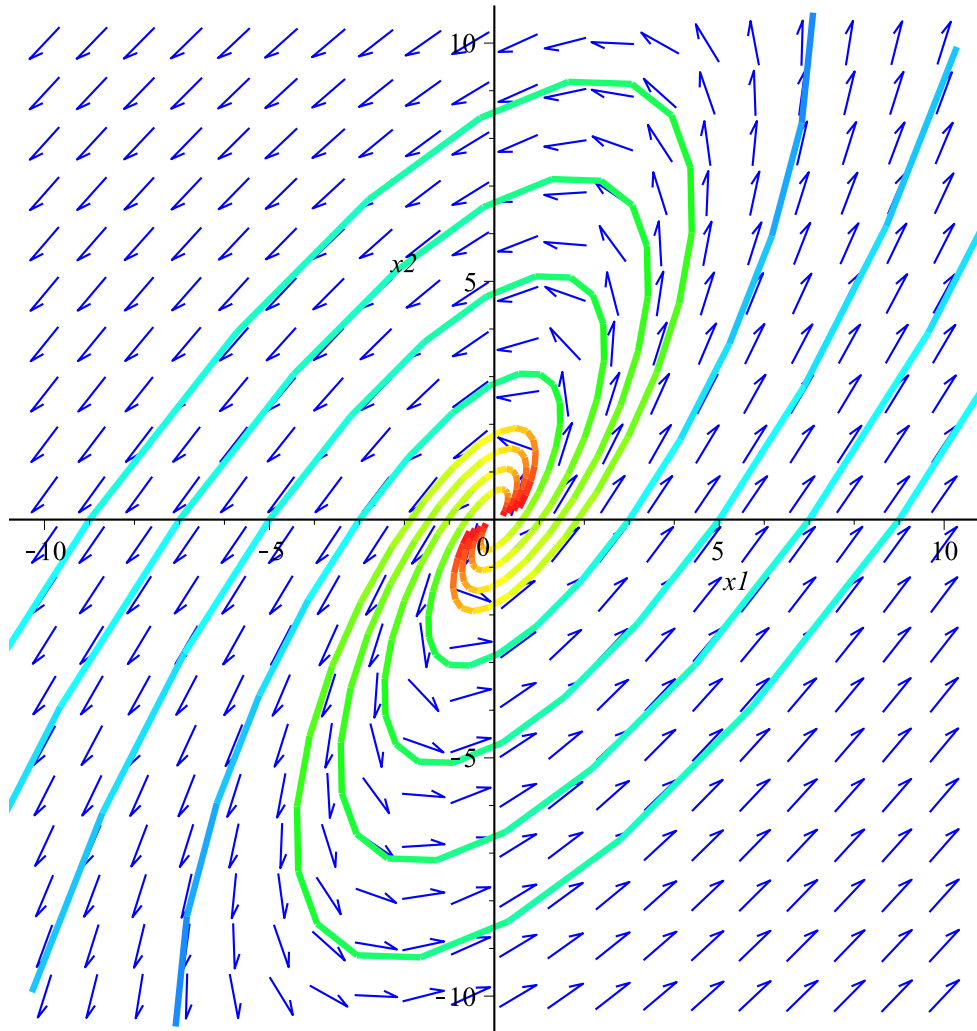
$$de5b := \frac{d}{dt} x2(t) = 4 x1(t) - x2(t)$$

(5)

```
> with(DEtools) :
```

```
> DEplot( {de5a, de5b}, [x1(t), x2(t)], t=-3..3, [[x1(0)=-5, x2(0)=0], [x1(0)=3, x2(0)=0], [x1(0)=5, x2(0)=0], [x1(0)=7, x2(0)=0], [x1(0)=9, x2(0)=0], [x1(0)=-3, x2(0)=0], [x1(0)=-9, x2(0)=0], [x1(0)=-7, x2(0)=0]], x1=-10..10, x2=-10..10, title='Unstable Spiral', color=blue, linecolor=t);
```

Unstable Spiral



```
> de6a := diff(x1(t), t) = 2·x1(t) - 5·x2(t);  
de6b := diff(x2(t), t) = x1(t) - 2·x2(t);
```

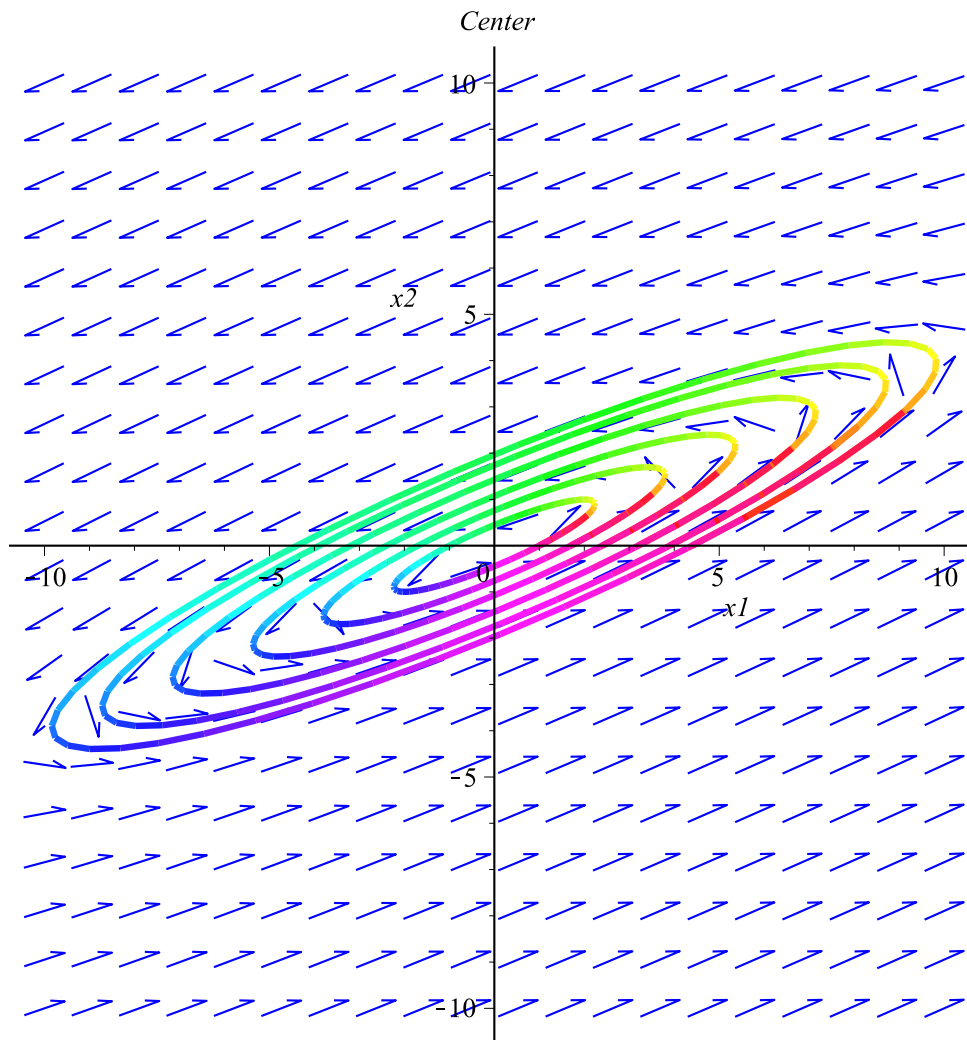
$$de6a := \frac{d}{dt} x1(t) = 2 x1(t) - 5 x2(t)$$

$$de6b := \frac{d}{dt} x2(t) = x1(t) - 2 x2(t)$$

(6)

```
> DEplot( {de6a, de6b}, [x1(t), x2(t)], t=0..7, [[x1(0) = 1, x2(0) = 0], [x1(0) = 1.7, x2(0)  
= 0], [x1(0) = 2.4, x2(0) = 0], [x1(0) = 3.2, x2(0) = 0], [x1(0) = 3.9, x2(0) = 0], [x1(0)  
= 4.4, x2(0) = 0]], x1=-10..10, x2=-10..10, title='Center', color=blue, linecolor=t);
```





>  $de7a := \text{diff}(x1(t), t) = 2 \cdot x1(t);$   
 $de7b := \text{diff}(x2(t), t) = 2 \cdot x2(t);$

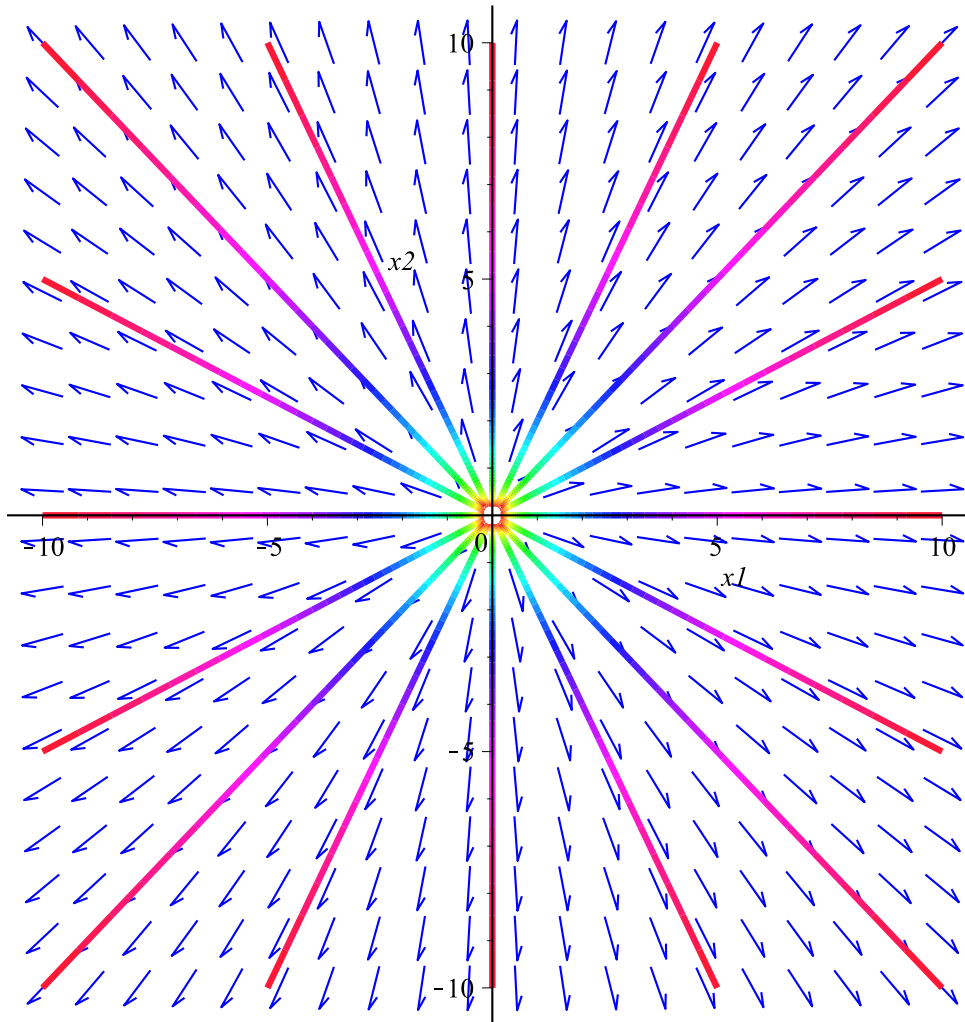
$$de7a := \frac{d}{dt} x1(t) = 2 x1(t)$$

$$de7b := \frac{d}{dt} x2(t) = 2 x2(t)$$

(7)

>  $DEplot(\{de7a, de7b\}, [x1(t), x2(t)], t = -2..0, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10, x2(0) = 5], [x1(0) = 10, x2(0) = 10], [x1(0) = 5, x2(0) = 10], [x1(0) = 0, x2(0) = 10], [x1(0) = -5, x2(0) = 10], [x1(0) = -10, x2(0) = 10], [x1(0) = -10, x2(0) = 5], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -5], [x1(0) = -10, x2(0) = -10], [x1(0) = -5, x2(0) = -10], [x1(0) = 0, x2(0) = -10], [x1(0) = 5, x2(0) = -10], [x1(0) = 10, x2(0) = -10], [x1(0) = 10, x2(0) = -5]], x1 = -10..10, x2 = -10..10, title = 'Unstable Proper Node', color = blue, linecolor = t);$

*Unstable Proper Node*



>  $de8a := \text{diff}(x1(t), t) = -x1(t) + x2(t);$   
 $de8b := \text{diff}(x2(t), t) = -x2(t);$

$$de8a := \frac{d}{dt} x1(t) = -x1(t) + x2(t)$$

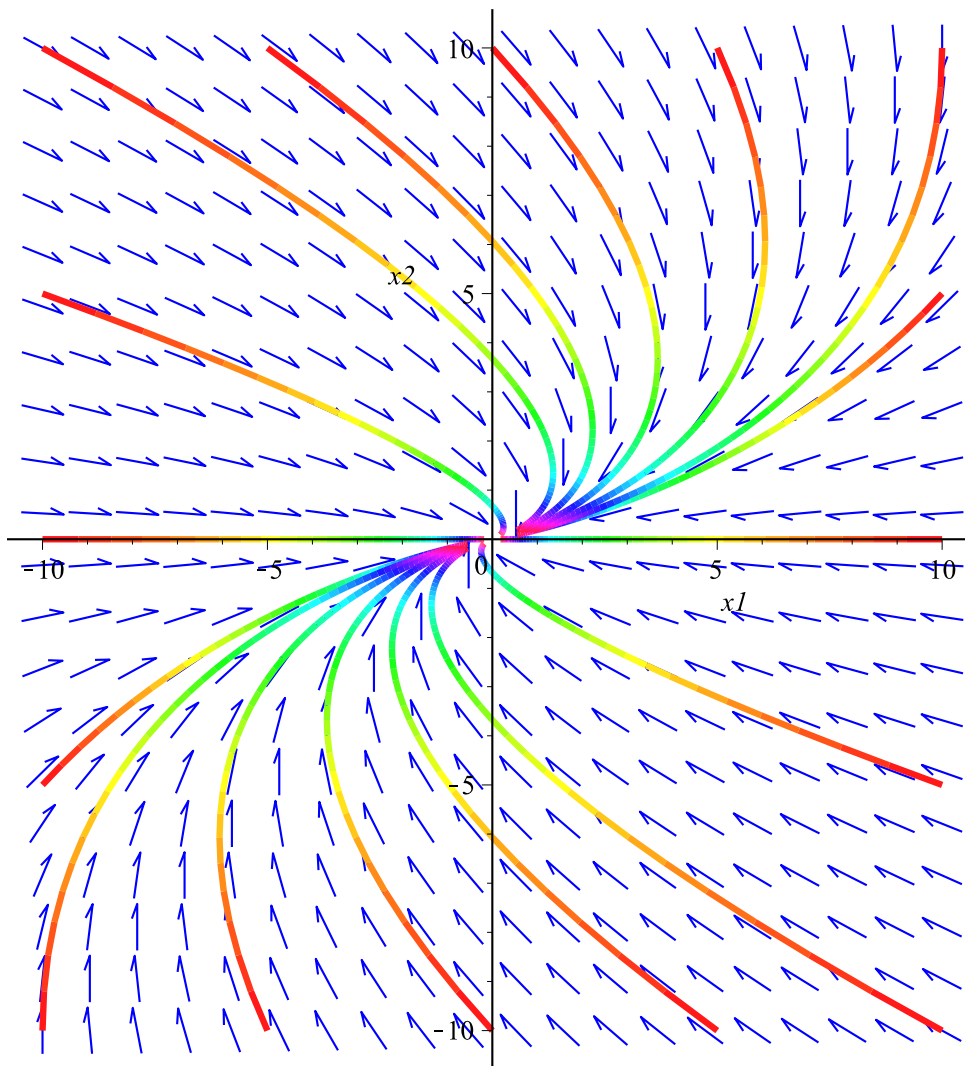
$$de8b := \frac{d}{dt} x2(t) = -x2(t)$$

(8)

>  $DEplot(\{de8a, de8b\}, [x1(t), x2(t)], t=0..4, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10, x2(0) = 5], [x1(0) = 10, x2(0) = 10], [x1(0) = 5, x2(0) = 10], [x1(0) = 0, x2(0) = 10], [x1(0) = -5, x2(0) = 10], [x1(0) = -10, x2(0) = 10], [x1(0) = -10, x2(0) = 5], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -5], [x1(0) = -10, x2(0) = -10], [x1(0) = -5, x2(0) = -10], [x1(0) = 0, x2(0) = -10], [x1(0) = 5, x2(0) = -10], [x1(0) = 10, x2(0) = -10], [x1(0) = 10, x2(0) = -5]], x1 = -10..10, x2 = -10..10, title = 'Stable Improper Node', color = blue,$

`linecolor = t);`

Stable Improper Node



`>`  
`>`

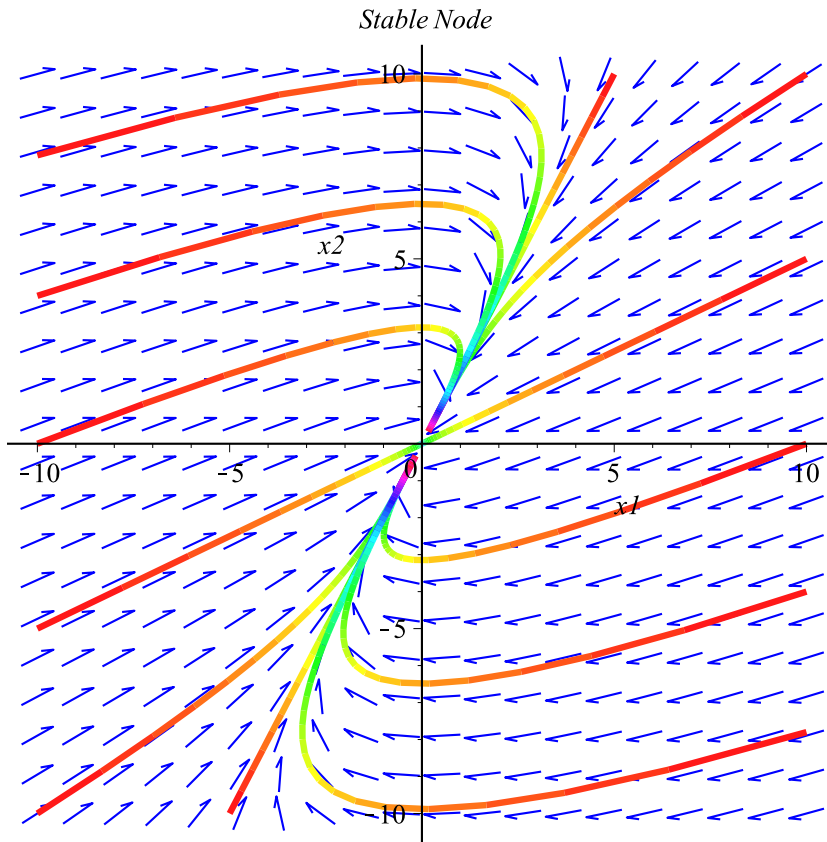
```
a := -5;  
deB1a := diff(x1(t), t) = a·x1(t) + 2· x2(t);  
deB1b := diff(x2(t), t) = -2· x1(t);
```

$$a := -5$$

$$deB1a := \frac{d}{dt} x1(t) = -5 x1(t) + 2 x2(t)$$

$$deB1b := \frac{d}{dt} x2(t) = -2 x1(t)$$

```
> DEplot( {deB1a, deB1b}, [x1(t), x2(t)], t=0..3, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10, x2(0) = 5], [x1(0) = 10, x2(0) = 10], [x1(0) = 5, x2(0) = 10], [x1(0) = -10, x2(0) = 7.8], [x1(0) = -10, x2(0) = 4], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -5], [x1(0) = -10, x2(0) = -10], [x1(0) = -5, x2(0) = -10], [x1(0) = 10, x2(0) = -7.8], [x1(0) = 10, x2(0) = -4]], x1 = -10..10, x2 = -10..10, title = 'Stable Node', color = blue, linecolor = t);
```



```
> a := -4;
deB2a := diff(x1(t), t) = a·x1(t) + 2·x2(t);
deB2b := diff(x2(t), t) = -2·x1(t);
```

$a := -4$

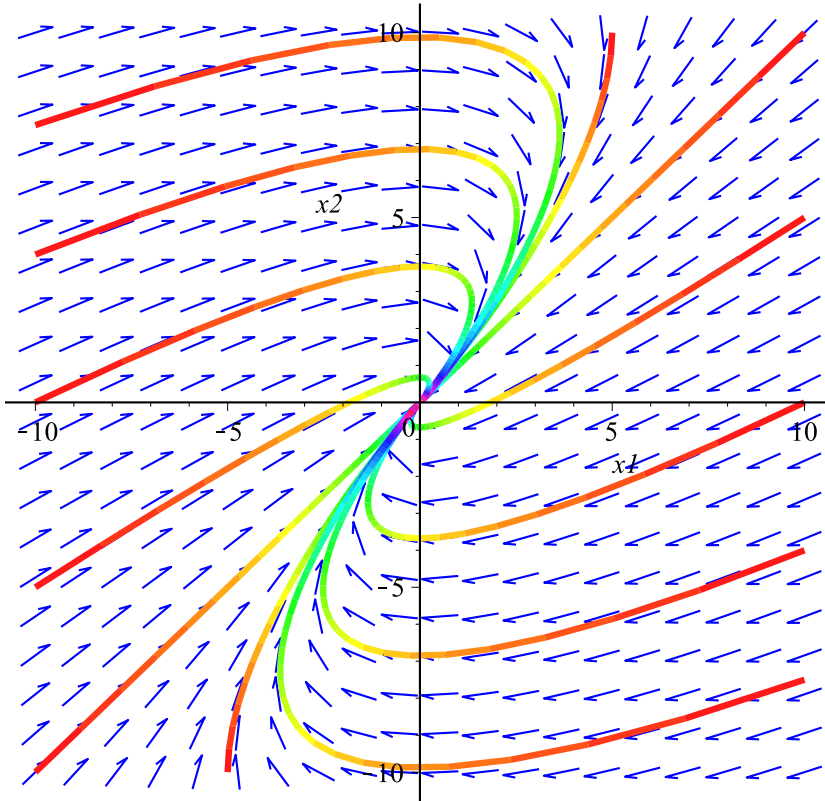
$$deB2a := \frac{d}{dt} x1(t) = -4 x1(t) + 2 x2(t)$$

$$deB2b := \frac{d}{dt} x2(t) = -2 x1(t)$$

**(10)**

```
> DEplot( {deB2a, deB2b}, [x1(t), x2(t)], t=0..3, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10, x2(0) = 5], [x1(0) = 10, x2(0) = 10], [x1(0) = 5, x2(0) = 10], [x1(0) = -10, x2(0) = 7.5], [x1(0) = -10, x2(0) = 4], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -5], [x1(0) = -10, x2(0) = -10], [x1(0) = -5, x2(0) = -10], [x1(0) = 10, x2(0) = -7.5], [x1(0) = 10, x2(0) = -4]], x1 = -10..10, x2 = -10..10, title = 'Stable Improper Node', color = blue, linecolor = t);
```

Stable Improper Node



```
> a := -2;
   deB3a := diff(x1(t), t) = a·x1(t) + 2·x2(t);
   deB3b := diff(x2(t), t) = -2·x1(t);
```

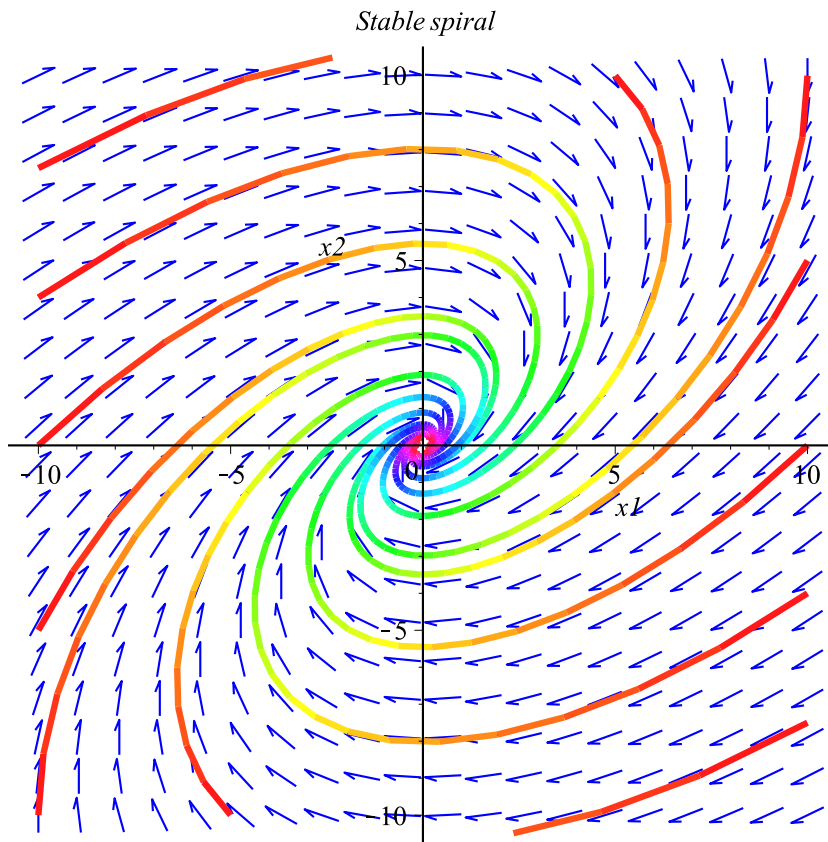
$$a := -2$$

$$deB3a := \frac{d}{dt} x1(t) = -2 x1(t) + 2 x2(t)$$

$$deB3b := \frac{d}{dt} x2(t) = -2 x1(t)$$

(11)

```
> DEplot( {deB3a, deB3b}, [x1(t), x2(t)], t=0..4, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10, x2(0) = 5], [x1(0) = 10, x2(0) = 10], [x1(0) = 5, x2(0) = 10], [x1(0) = -10, x2(0) = 7.5], [x1(0) = -10, x2(0) = 4], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -5], [x1(0) = -10, x2(0) = -10], [x1(0) = -5, x2(0) = -10], [x1(0) = 10, x2(0) = -7.5], [x1(0) = 10, x2(0) = -4]], x1 = -10..10, x2 = -10..10, title = 'Stable spiral', color = blue, linecolor = t);
```



```
> a := 0;
deB4a := diff(x1(t), t) = a·x1(t) + 2·x2(t);
deB4b := diff(x2(t), t) = -2·x1(t);
```

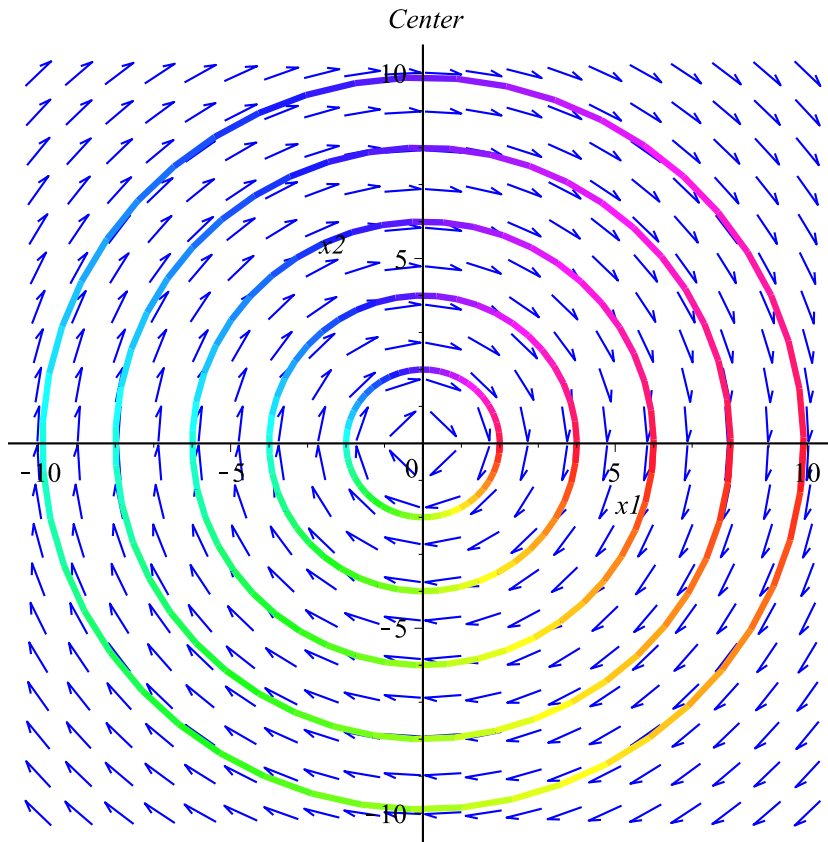
$a := 0$

$$deB4a := \frac{d}{dt} x1(t) = 2 x2(t)$$

$$deB4b := \frac{d}{dt} x2(t) = -2 x1(t)$$

**(12)**

```
> DEplot( {deB4a, deB4b}, [x1(t), x2(t)], t=0..3.2, [[x1(0)=2, x2(0)=0], [x1(0)=4, x2(0)=0], [x1(0)=6, x2(0)=0], [x1(0)=8, x2(0)=0], [x1(0)=9.9, x2(0)=0]], x1=-10..10, x2=-10..10, title='Center', color=blue, linecolor=t);
```



```
> a := 2;
deB5a := diff(x1(t), t) = a·x1(t) + 2·x2(t);
deB5b := diff(x2(t), t) = -2·x1(t);
```

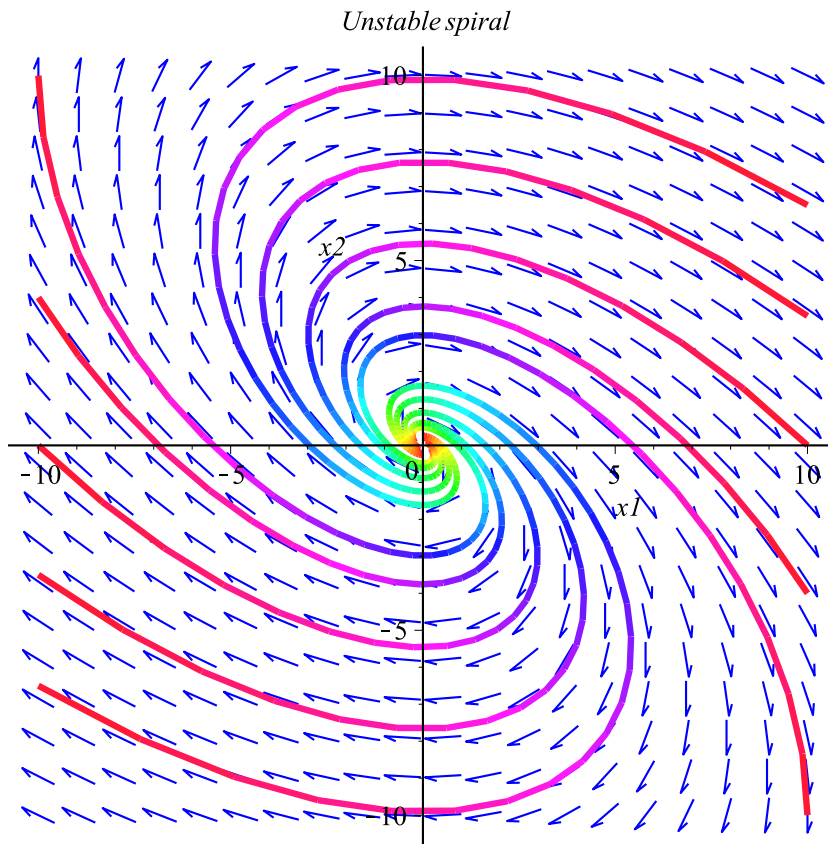
$a := 2$

$$deB5a := \frac{d}{dt} x1(t) = 2 x1(t) + 2 x2(t)$$

$$deB5b := \frac{d}{dt} x2(t) = -2 x1(t)$$

**(13)**

```
> DEplot( {deB5a, deB5b}, [x1(t), x2(t)], t=0..-4, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10,
x2(0) = 3.5], [x1(0) = 10, x2(0) = 6.5], [x1(0) = -10, x2(0) = 10], [x1(0) = -10, x2(0)
= 4], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -3.5], [x1(0) = -10, x2(0) = -6.5],
[x1(0) = 10, x2(0) = -10], [x1(0) = 10, x2(0) = -4]], x1 = -10..10, x2 = -10..10, title =
'Unstable spiral', color = blue, linecolor = t);
```



```
> a := 4;
   deB6a := diff(x1(t), t) = a·x1(t) + 2·x2(t);
   deB6b := diff(x2(t), t) = -2·x1(t);
```

```
      a := 4
```

$$deB6a := \frac{d}{dt} x1(t) = 4 x1(t) + 2 x2(t)$$

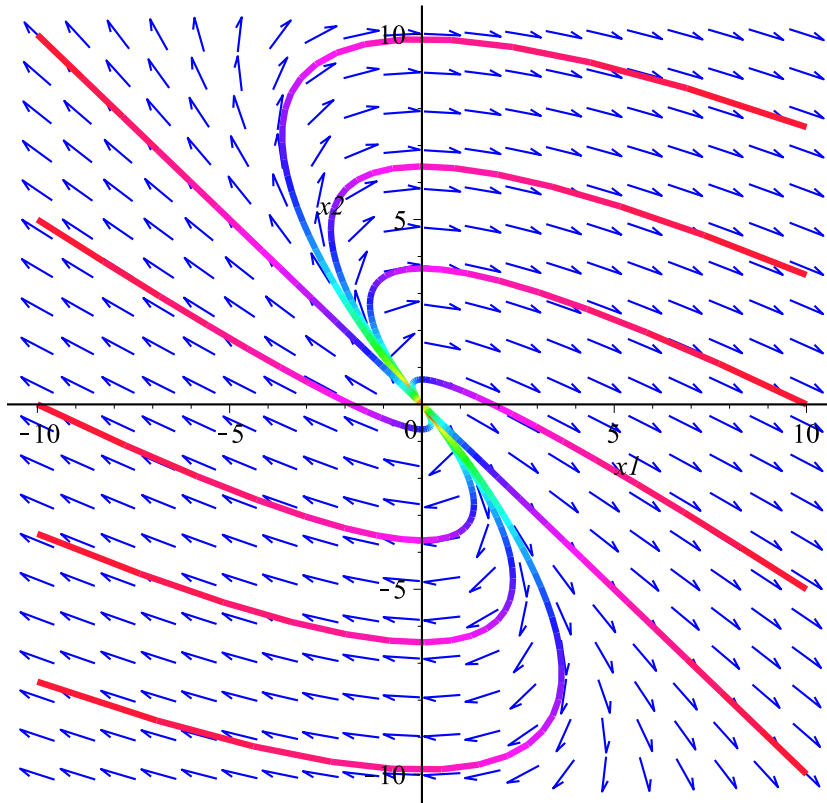
$$deB6b := \frac{d}{dt} x2(t) = -2 x1(t)$$

**(14)**

```
> DEplot( {deB6a, deB6b}, [x1(t), x2(t)], t=0..-3, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10,
x2(0) = 3.5], [x1(0) = 10, x2(0) = 7.5], [x1(0) = -10, x2(0) = 10], [x1(0) = -10, x2(0)
= 5], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -3.5], [x1(0) = -10, x2(0) = -7.5],
[x1(0) = 10, x2(0) = -10], [x1(0) = 10, x2(0) = -5]], x1 = -10..10, x2 = -10..10, title =
'Unstable Improper Node', color = blue, linecolor = t);
```



Unstable Improper Node



```
> a := 5;
   deB7a := diff(x1(t), t) = a·x1(t) + 2·x2(t);
   deB7b := diff(x2(t), t) = -2·x1(t);
```

```
      a := 5
```

$$deB7a := \frac{d}{dt} x1(t) = 5 x1(t) + 2 x2(t)$$

$$deB7b := \frac{d}{dt} x2(t) = -2 x1(t)$$

(15)

```
> DEplot( {deB7a, deB7b}, [x1(t), x2(t)], t=0..-3, [[x1(0) = 10, x2(0) = 0], [x1(0) = 10,
x2(0) = 3.5], [x1(0) = 10, x2(0) = 7.5], [x1(0) = -5, x2(0) = 10], [x1(0) = -10, x2(0)
= 10], [x1(0) = -10, x2(0) = 5], [x1(0) = -10, x2(0) = 0], [x1(0) = -10, x2(0) = -3.5],
[x1(0) = -10, x2(0) = -7.5], [x1(0) = 5, x2(0) = -10], [x1(0) = 10, x2(0) = -10], [x1(0)
= 10, x2(0) = -5]], x1 = -10..10, x2 = -10..10, title = 'Unstable Node', color = blue, linecolor
= t);
```

*Unstable Node*

