1. a. The 3 populations are $p_1 = 700$, $p_2 = 860$, and $p_3 = 988$.

b. The equilibrium is $p_e = 1500$. The equilibrium is stable.

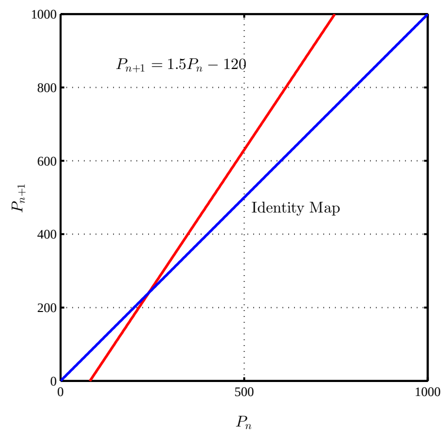
2. a. The breathing fraction is $q = 0.120536$, and the functional reserve capacity is $V_r = 2188.9$ ml.

b. The concentration of Helium in the next two breaths are $c_2 = 39.85$ and $c_3 = 35.67$. The equilibrium concentration is $c_e = \gamma = 5.2$ ppm of He, which is a stable equilibrium.

3. a. For the Malthusian growth model with dispersion, $P_{n+1} = (1+r)P_n - \mu$, $r = 0.5$ and $\mu = 120$. The populations in the next two weeks ar $P_3 = 1117.5$ and $P_4 = 1556.25$.

b. The equilibrium is $P_e = 240$, and it is unstable.

c. The graph of the updating function and identity map, $P_{n+1} = P_n$, are shown below. The only point of intersection occurs at the equilibrium found above.



4. a. From the breathing model, $c_{n+1} = (1-q)c_n + q\gamma$ and the data $c_0 = 400$, $c_1 = 352$, and $c_2 = 310$, we find the constants q and γ by substitution and the simultaneous solution of two equations and two unknowns. We have

$$352 = 400(1-q) + q\gamma \qquad \text{and} \qquad 310 = 352(1-q) + q\gamma.$$

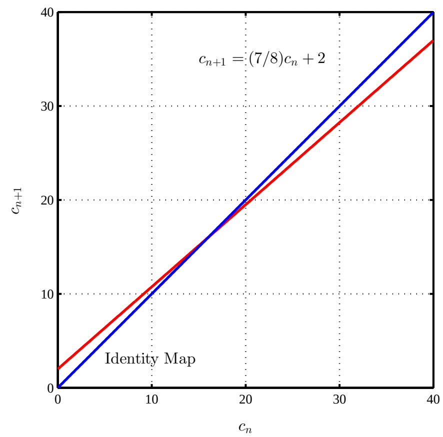
Subtracting the second equation from the first gives $42 = 48(1-q)$ or $1 - q = \frac{42}{48} = \frac{7}{8}$. Thus, $q = \frac{1}{8}$. This value is substituted into the first equation above to give $352 = 400\frac{7}{8} + \frac{1}{8}\gamma$, which gives $\gamma = 16$.

Thus, the model becomes $c_{n+1} = \frac{7}{8}c_n + 2$, and the next 2 breaths satisfy

$$\begin{aligned} c_3 &= \frac{7}{8}(310) + 2 = 273.25 \\ c_4 &= \frac{7}{8}(273.25) + 2 = 241.1 \end{aligned}$$

b. At the equilibria, $c_e = \frac{7}{8}c_e + 2$, so $\frac{1}{8}c_e = 2$ or $c_e = 16$, which is the value of γ as expected. This equilibrium is stable.

c. The graph of the updating function and identity map, $c_{n+1} = c_n$, are shown below. The only point of intersection occurs at the equilibrium, γ found above.

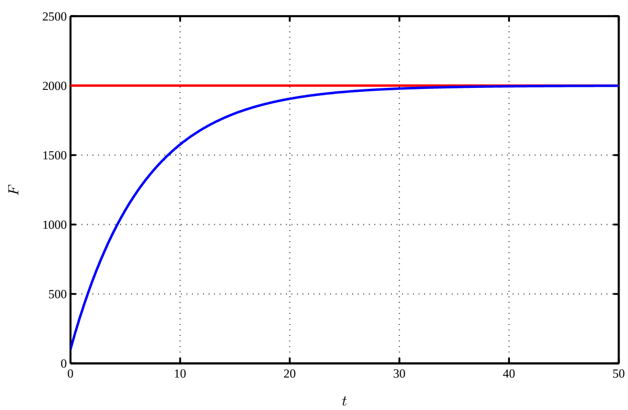


5. a. The next two years satisfy

$$F_1 = 0.86(100) + 280 = 366 \qquad \text{and} \qquad F_2 = 0.86(366) + 280 = 594.8.$$

At equilibrium, $F_e = 0.86\,F_e + 280$ or $F_e = 2000$. This is a stable equilibrium. (The slope $a = 0.86 < 1$.)

b. The F-intercept is 100, and there is a horizontal asymptote at $F = 2000$. Below is the graph of this function.



c. Since $F(6) = 1227.5176$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

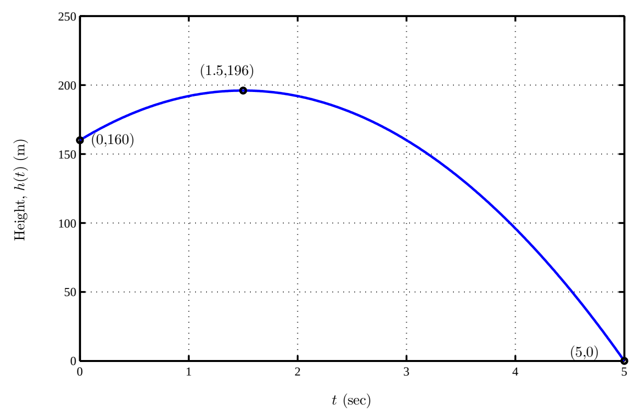
$$\frac{F(6) - F(5)}{6 - 5} = 125.01.$$

Since $F(5.1) = 1115.8655$ and $F(5) = 1102.50355$, then the slope of the secant line is given by

$$\frac{F(5.1) - F(5)}{5.1 - 5} = 133.62.$$

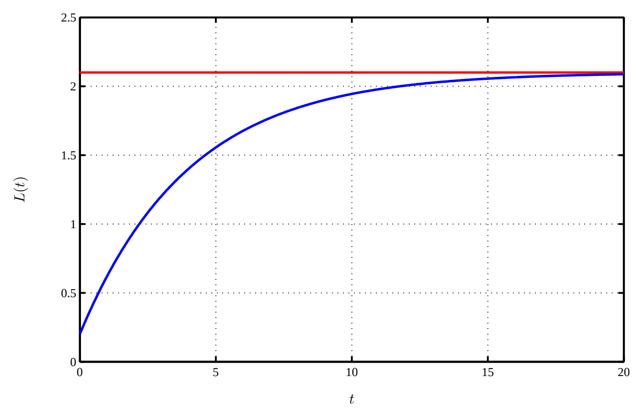
6. a. The average velocity over the for $t \in [0,2]$ is 16 ft/sec. The average velocity over the for $t \in [1,1.2]$ is 12.8 ft/sec. The average velocity over the for $t \in [1,1.01]$ is 15.84 ft/sec.

b. The ball hits the ground at 5 sec with an approximate velocity of $v_{ave} = \frac{h(5)-h(4.999)}{0.001} = -111.984$ ft/sec. The graph is below.



7. a. Asymptotically, the leopard shark can reach 2.1 m. The length of the leopard shark at birth is 0.2 m, at 1 yr is 0.62 m, at 5 yr is 1.56 m, and at 10 yr is 1.94 m. The maximum length is 2.1 m. The shark reaches 90% of its maximum length at $t = 8.81$ yr. The graph is below.

b. The average growth rate for $t \in [1,5]$ is $g_{ave} = 0.2338$ m/yr. The average growth rate for $t \in [5,10]$ is $g_{ave} = 0.07768$ m/yr. The average growth rate for $t \in [5,6]$ is $g_{ave} = 0.1204$ m/yr. The average growth rate for $t \in [5,5.01]$ is $g_{ave} = 0.1359$ m/yr. This last approximation is the best approximation to the derivative (which has the value of $L'(5) = 0.1361$ m/yr).



8. a. The serval can catch any bird flying at heights from 16 to 25 ft or up to 9 ft above the serval.

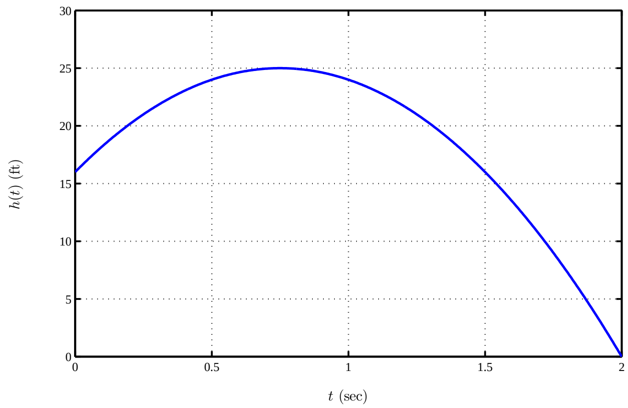
b. The average velocity of the serval for $t \in [0, \frac{1}{4}]$ is $v_{ave} = 20$ ft/sec. The average velocity of the serval for $t \in [\frac{1}{2}, 1]$ is $v_{ave} = 0$ ft/sec. The average velocity of the serval for $t \in [1, \frac{5}{4}]$ is $v_{ave} = -12$ ft/sec.

c. The velocity satisfies:

$$v(t) = h'(t) = 24 - 32\,t.$$

Thus, $v(1) = -8$ ft/sec.

d. The serval hits the ground at $t = 2$. The velocity when it hits the ground is $v(2) = -40$ ft/sec. A graph of the height of the serval is below.



9. a. The vertical velocity is $v_0 = 420\sqrt{2} \simeq 593.97$ cm/sec. The impala is in the air for $t = \frac{6\sqrt{2}}{7} \simeq 1.21218$ sec.

b. The average velocity for the impala between $t = 0$ and $t = 0.5$ is $v_{ave} = 420\sqrt{2} - 245 \simeq 348.97$ cm/sec.

10. a. The slope of the secant line is

$$m(h) = \frac{f(2+h) - f(2)}{h} = \frac{\frac{2+h-2}{2(2+h)+2} - 0}{h} = \frac{1}{6+2h}.$$

b. The slope of the tangent line

$$\lim_{h \to 0} \frac{1}{6+2h} = \frac{1}{6}.$$

The equation of the tangent line is

$$y - 0 = \frac{1}{6}(x - 2) \quad \text{or} \quad y = \frac{1}{6}x - \frac{1}{3}.$$

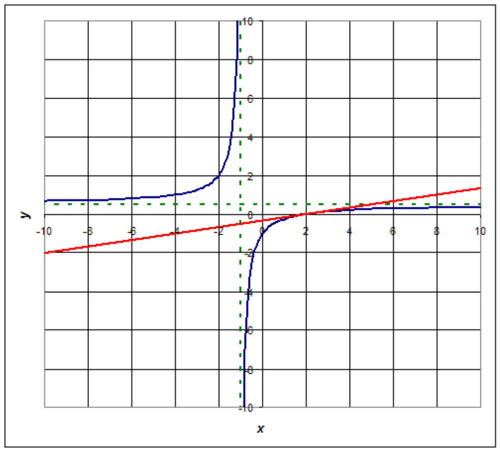
c. The x-intercept is $x = 2$, and the y-intercept is $y = -1$. There is a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = \frac{1}{2}$. Below is the graph of the function and the tangent line.

11. a. Write $f(x)$ as powers of x as much as possible (remove denominators), so

$$f(x) = 6x^3 + 2x^{-2} - e^{2x}(x^2 - 9).$$

Apply power rules, product rule, and the rules for exponential yielding

$$\begin{aligned} f'(x) &= 6(3x^2) + 2(-2x^{-3}) - \left(e^{2x}(2x) + 2e^{2x}(x^2 - 9)\right) \\ &= 18x^2 - \frac{4}{x^3} - 2e^{2x}(x^2 + x - 9) \end{aligned}$$



Problem 10

b. Use the properties of logarithms to write

$$g(x) = 2e^{-3x} + 2\ln(x) - 5.$$

Use the rules of differentiation of exponentials and logarithms to give

$$\begin{aligned} g'(x) &= 2(-3)e^{-3x} + \frac{2}{x} \\ &= \frac{2}{x} - 6e^{-3x} \end{aligned}$$

c. Leave $h(x)$ in the form,

$$h(x) = 2x^6 \ln(x) - e^{\sin(2x)} + \frac{1}{2}e^{-4x}.$$

Apply power rules, product rule, chain rule, and the rules for exponentials and logarithms yielding

$$\begin{aligned} h'(x) &= 2\left((6x^5)\ln(x) + x^6\left(\frac{1}{x}\right)\right) - e^{\sin(2x)}(2\cos(2x)) + \frac{-4}{2}e^{-4x} \\ &= 12x^5\ln(x) + 2x^5 - 2\cos(2x)e^{\sin(2x)} - 2e^{-4x} \end{aligned}$$

d. Given:

$$b(x) = \ln(\cos(3x)) - e^{x^2+4x}.$$

Apply power rule, chain rule, and the rules for exponentials and logarithms yielding

$$b'(x) = -\frac{3\sin(3x)}{\cos(3x)} - e^{x^2+4x}(2x+4).$$

e. Write

$$q(x) = \frac{2 + e^{2x}}{x^2 - 3} - (x^2 - \sin^3(x^2))^4.$$

Apply power rule, quotient rule, chain rule, and the rules for exponentials and trig functions yielding

$$q'(x) = \frac{(x^2-3)(2e^{2x}) - (2+e^{2x})(2x)}{(x^2-3)^2} - 4(x^2 - \sin^3(x^2))^3(2x - 6x\sin^2(x^2)\cos(x^2)).$$

f. Write $k(t)$ in the following form:

$$k(t) = \frac{1}{4}t^2 - 4(\cos(t^2+2))^{-1} + 4t^{-\frac{1}{2}}.$$

Apply power rules, the chain rule, and trig function rule yielding

$$\begin{aligned} k'(t) &= \frac{1}{2}t + 4(\cos(t^2+2))^{-2}(-\sin(t^2+2))2t - 2t^{-\frac{3}{2}} \\ &= \frac{1}{2}t - \frac{8t\sin(t^2+2)}{(\cos(t^2+2))^2} - 2t^{-\frac{3}{2}} \end{aligned}$$

g. Write $r(x)$ as follows:

$$r(x) = e^{2x}(x^3 - 5x + 7)^4 - e^{-x}\cos(2x).$$

Apply the product and chain rules with rules for exponentials and cosine to obtain

$$\begin{aligned} r'(x) &= \left(e^{2x}4(x^3-5x+7)^3(3x^2-5) + 2e^{2x}(x^3-5x+7)^4\right) \\ &\quad + e^{-x}(2\sin(2x) + \cos(2x)). \end{aligned}$$

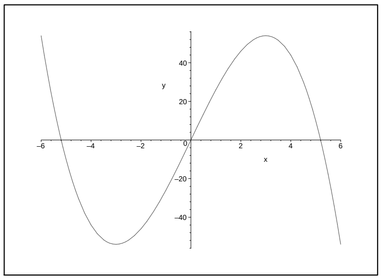
h. Write as

$$w(x) = \frac{x^4 + e^{-2x}}{x^3 + \cos(4x)} + 7x(x^2+2x+5)^{-\frac{1}{2}}.$$

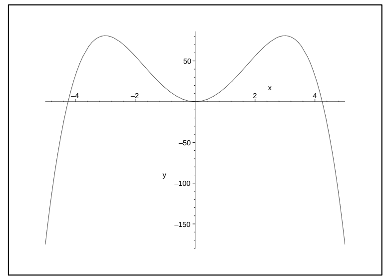
Apply the quotient, product, and chain rules:

$$\begin{aligned} w'(x) &= \frac{(x^3+\cos(4x))(4x^3 - 2e^{-2x}) - (x^4+e^{-2x})(3x^2 - 4\sin(4x))}{(x^3+\cos(4x))^2} \\ &\quad - \frac{7x}{2}(x^2+2x+5)^{-\frac{3}{2}}(2x+2) + 7(x^2+2x+5)^{-\frac{1}{2}}. \end{aligned}$$

12. a. $y = 27x - x^3$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0,0)$.
x-intercepts: $27x - x^3 = x(27 - x^2) = 0$, so $x = 0$ and $x = \pm\sqrt{27} = \pm 3\sqrt{3}$.
No asymptotes
Derivative $y'(x) = 27 - 3x^2$
Extrema are where $y'(x) = -3(x^2 - 9) = 0$, so $x = \pm 3$. With $y(-3) = 27(-3) - (-3)^3 = -54$ and $y(3) = 54$. Thus, $(3, 54)$ is a maximum, and $(-3, -54)$ is a minimum.
Second derivative $y''(x) = -3(2)x = -6x$.
Point of inflection ($y'' = 0$): At $x = 0$ or $(0, 0)$.



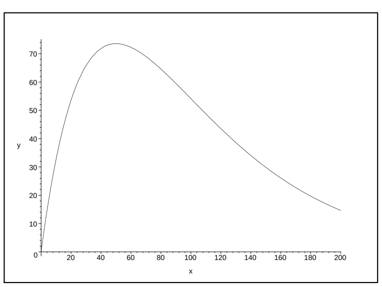
Problem 12a



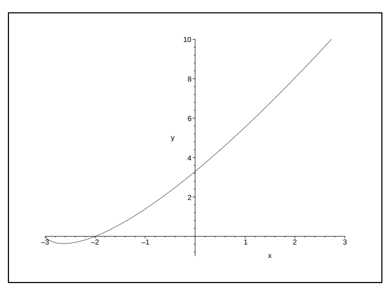
Problem 12b

b. $y = 18x^2 - x^4$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$.
x-intercept: $x^2(18 - x^2) = -x^2(x + 3\sqrt{2})(x - 3\sqrt{2}) = 0$, so $x = 0$ and $x = \pm 3\sqrt{2}$.
No asymptotes
Derivative $y'(x) = 36x - 4x^3 = 4x(9 - x^2)$
Critical points satisfy $y'(x) = -4x(x^2 - 9) = 0$, so $x = 0, \pm 3$. With $y(0) = 0$, $(0, 0)$ is a minimum.
When $x = \pm 3$,$y(\pm 3) = 81$, so there are local maxima at $(-3, 81)$ and $(3, 81)$.
Second derivative $y''(x) = 36 - 12x^2 = 12(3 - x^2)$.
Point of inflection ($y'' = 0$): At $x = \pm\sqrt{3}$, giving $(\pm\sqrt{3}, 45)$.

c. $y = 4xe^{-0.02x}$
Domain is all x.
y-intercept: $y(0) = 0$, so $(0, 0)$, which is also, the only x-intercept.
Horizontal asymptote: As $x \to \infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the right).
Derivative: By the product rule, $y'(x) = 4x(-0.02)e^{-0.02x} + 4e^{-0.02x} = 4e^{-0.02x}(1 - 0.02x)$
Critical points satisfy $y'(x) = 0$, so $1 - 0.02x = 0$ or $x = 50$. With $y(50) = 200e^{-1} \simeq 73.576$, $(50, 73.576)$ is a maximum.
Second derivative $y''(x) = 4e^{-0.02x}(-0.02) + 4(-0.02)e^{-0.02x}(1 - 0.02x) = -0.16(1 - 0.01x)e^{-0.02x}$.
Point of inflection ($y'' = 0$): At $x = 100$, $y(100) = 400e^{-2} \simeq 54.134$. Thus, $(100, 54.134)$.



Problem 12c

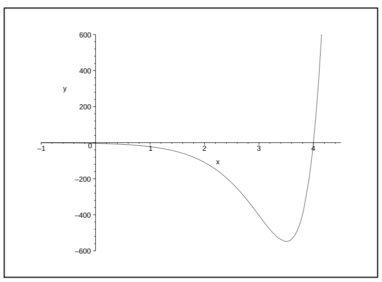


Problem 12d

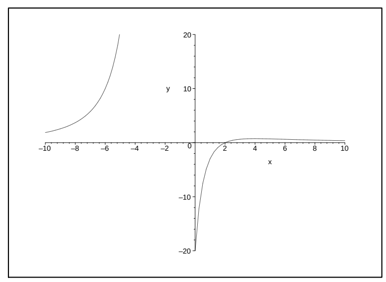
d. $y = (x + 3)\ln(x + 3)$
Domain is $x > -3$. The y-intercept is $3\ln(3) \simeq 3.2958$.
x-intercept: Where $(x + 3)\ln(x + 3) = 0$, which occurs when $\ln(x + 3) = 0$ or $x = -2$.
There are no asymptotes. (It can be shown that as $x \to -3$, $y \to 0$.)

Derivative: By the product rule, $y'(x) = \frac{x+3}{x+3} + \ln(x+3) = 1 + \ln(x+3)$.
Critical points satisfy $y'(x) = 0$, so $\ln(x+3) = -1$ or $x+3 = e^{-1} \simeq 0.3679$, so $x \simeq -2.6321$. When $x = e^{-1} - 3$, $y = -e^{-1}$ and is a minimum.
Second derivative $y''(x) = \frac{1}{x+3} > 0$ for $x > -3$. There is no point of inflection, and the function is concave up.

e. $y = (x-4)e^{2x}$
Domain is all x.
y-intercept: $y(0) = -4$, so $(0, -4)$.
x-intercept: Since the exponential function is not zero, $y = 0$ when $x = 4$.
Horizontal asymptote: As $x \to -\infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the left).
Derivative: By the product rule, $y'(x) = 2(x-4)e^{2x} + e^{2x} = (2x-7)e^{2x}$.
Critical points satisfy $y'(x) = 0$, so $2x - 7 = 0$ or $x = 3.5$. With $y(3.5) = -0.5e^7 \simeq -548.3$, $(3.5, -548.3)$ is a minimum.
Second derivative $y''(x) = 2(2x-7)e^{2x} + 2e^{2x} = 4(x-3)e^{2x}$.
Point of inflection ($y'' = 0$): At $x = 3$, $y(3) = -e^6 \simeq -403.4$. Thus, $(3, -402.4)$.



Problem 12e

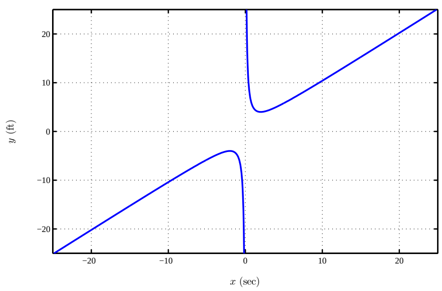


Problem 12f

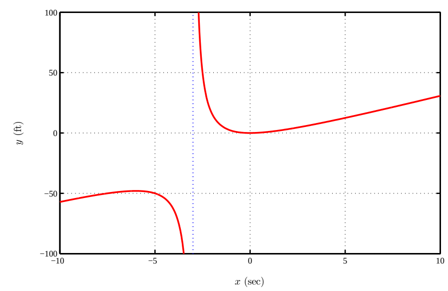
f. $y = \dfrac{10(x-2)}{(1+0.5x)^3}$
Domain is all $x \neq -2$.
y-intercept: $y(0) = -20$, so $(0, -20)$.
x-intercept: Numerator equal to zero, so $x = 2$ or $(2, 0)$
Vertical asymptote: $x = -2$.
Horizontal asymptote: The power of the denominator exceeds the power of the numerator, so $y = 0$ is a horizontal asymptote
Derivative: By the quotient rule, $y'(x) = 10\frac{(1+0.5x)^3 - (x-2)3(1+0.5x)^2(0.5)}{(1+0.5x)^6} = \frac{10(4-x)}{(1+0.5x)^4}$.
Critical points satisfy $y'(x) = 0$, so $4 - x = 0$ or $x = 4$. With $y(4) = \frac{20}{27} \simeq 0.7407$, $(4, 0.7407)$ is a relative maximum.
Second derivative $y''(x) = 10\frac{-(1+0.5x)^4 - (4-x)4(1+0.5x)^3(0.5)}{(1+0.5x)^8} = \frac{15(x-6)}{(1+0.5x)^5}$.
Since $y''(x) = 0$ and $x = 6$, there is a point of inflection at $(6, \frac{5}{8})$.

g. $y = x + \dfrac{4}{x} = x + 4x^{-1}$
Domain is all $x \neq 0$.
Since there is a vertical asymptote at $x = 0$, there is no y-intercept.
We solve $y = \frac{x^2+4}{x} = 0$ or $x^2 + 4 = 0$, so no x-intercepts.

Derivative $y'(x) = 1 - 4x^{-2} = \frac{x^2-4}{x^2}$
Critical points satisfy $y'(x) = 0$, so $x^2 - 4 = 0$ or $x = \pm 2$. With $y(-2) = -4$, $(-2, -4)$ is a local maximum. With $y(2) = 4$, $(2, 4)$ is a local minimum.
Second derivative $y''(x) = 8x^{-3}$, which is never zero, so no points of inflection.



Problem 12g



Problem 12h

h. $y = \dfrac{4x^2}{x+3}$
Domain all $x \neq -3$
x and y-intercept: $(0, 0)$.
Vertical asymptote: $x = -3$
Derivative: By the quotient rule, $y'(x) = \frac{4(2x(x+3)-x^2)}{(x+3)^2} = \frac{4x(x+6)}{(x+3)^2}$.
Critical points satisfy $y'(x) = 0$, so $x = 0$ and $x = -6$. When $x = 0$, $y = 0$ and is a minimum. When $x = -6$, $y = -48$ and is a maximum.
Second derivative $y''(x) = \frac{4((x^2+6x+9)(2x+6)-(x^2+6x)(2x+6))}{(x+3)^4} = \frac{36(2x+6)}{(x+3)^4}$. There is no point of inflection, as $y''(x) = 0$ at $x = -3$, the vertical asymptote.

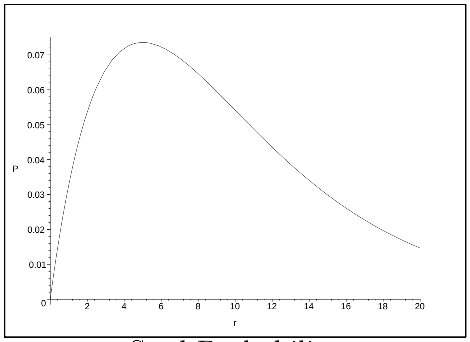
13. a. The temperature is given by $T(t) = 0.002t^3 - 0.09t^2 + 1.2t + 32$, which upon differentiation becomes

$$\frac{dT}{dt} = 0.006\,t^2 - 0.18\,t + 1.2.$$

At noon, $T'(12) = 0.006(144) - 0.18(12) = -0.096$ °C/hr.

b. To find extrema, solve $T'(t) = 0.006(t^2 - 30t + 2000) = 0.006(t-10)(t-20) = 0$. It follows $t = 10$ and $t = 20$, so $T(10) = 2 - 9 + 12 + 32 = 37$ and $T(20) = 16 - 36 + 24 + 32 = 36$. The maximum temperature of the subject occurs at 10 AM with a temperature of 37 °C, while the minimum temperature of the subject occurs at 8 PM ($t = 20$) with a temperature of 36 °C.

14. By the product rule, the derivative is $P'(r) = 0.04e^{-0.2r} - 0.008re^{-0.2r}$.The maximum probability occurs when the derivative is zero, $0.04e^{-0.2r} - 0.008re^{-0.2r} = 0.04e^{-0.2r}(1 - 0.2r)$ or $0.2r = 1$. Thus, the maximum probability of a seed landing occurs at $r = 5$ m with a probability of $P(5) = 0.0736$. The graph of the probability density function has an intercept at $(0, 0)$ ($P(0) = 0$), a horizontal asymptote of $P = 0$ (since for large r, P becomes arbitrarily small), and a local maximum of $(5, 0.0736)$.
15. a. The equilibrium satisfies $N_e(0.8 - 0.04\ln(N_e)) = 0$. Since $N = 0$ is not in the domain. Thus, the equilibrium satisfies $0.04\ln(N_e) = 0.8$ or $\ln(N_e) = 20$. It follows that the equilibrium is $N_e = 4.852 \times 10^8$.



Seed Probability

b. By the product rule, the derivative is $G'(N) = -N(0.04/N) + (0.8 - 0.04\ln(N)) = 0.76 - 0.04\ln(N)$. The maximum growth rate satisfies $0.76 - 0.04\ln(N) = 0$ or $\ln(N) = 19$. Thus, the maximum rate of growth occurs at $N_{max} = e^{19} = 1.785 \times 10^8$ with a maximum growth rate of $G(N_{max}) = 7.139 \times 10^6$.

c. Evaluating $G(2 \times 10^8) = 7.089 \times 10^6$, so the tumor is growing with this population of cells. Evaluating $G'(2 \times 10^8) = -0.004553$, so the rate of growth of the tumor is decreasing with this population of cells.

16. a. The concentration of glucose is given by $g(t) = 80 + 150e^{-0.8t}\sin(t)$, so for it to reach 80 mg/100 ml of blood after $t > 0$, we need $80 = 80 + 150e^{-0.8t}\sin(t)$ or $0 = \sin(t)$ or $t = n\pi, \quad n = 0, 1, ...$ The next time is $t_1 = \pi \approx 3.14$ hr.

b. The rate of change of glucose per hour is

$$\frac{dg}{dt} = 150\left((-0.8)e^{-0.8t}\sin(t) + e^{-0.8t}\cos(t)\right) = 150\,e^{-0.8t}(\cos(t) - 0.8\sin(t)).$$

At $t = 1$, $g'(1) = 150\,e^{-0.8}(\cos(1) - 0.8\sin(1)) = -8.9557$ mg/100 ml of blood/hour. To find the absolute maximum, we solve $g'(t_{max}) = 0$, so

$$\begin{aligned} 150\,e^{-0.8t_{max}}(\cos(t_{max}) - 0.8\sin(t_{max})) &= 0, \\ \cos(t_{max}) &= 0.8\sin(t_{max}), \\ \tan(t_{max}) &= 1.25, \\ t_{max} &= \arctan(1.25) \approx 0.8961 \text{ hr.} \end{aligned}$$

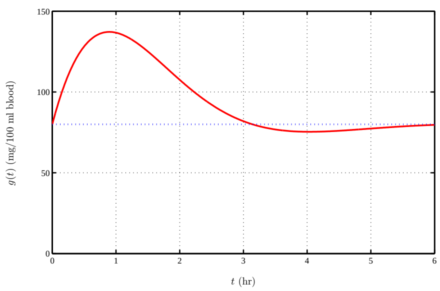
The absolute maximum is $g(t_{max}) = 137.19$ mg/100 ml of blood. The absolute minimum occurs at $t_{min} = t_{max} + \pi = 4.0376$ with $g(t_{min}) = 75.367$ mg/100 ml of blood. The graph for the concentration of glucose in the blood is below.

c. The level of insulin satisfies the function $i(t) = 10(e^{-0.4t} - e^{-0.5t})$, so

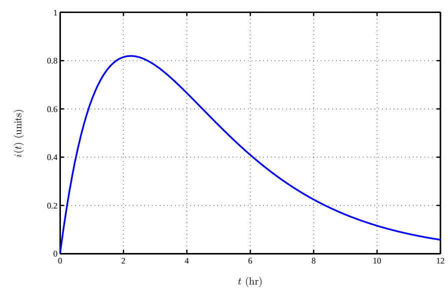
$$i'(t) = 10(-0.4e^{-0.4t} + 0.5e^{-0.5t}) = 5e^{-0.5t} - 4e^{-0.4t}.$$

The concentration is maximum where $i'(t) = 0$, so $5e^{-0.5t} = 4e^{-0.4t}$ or $\frac{5}{4} = e^{-0.4t}e^{0.5t} = e^{0.1t}$. It follows that $t = 10\ln\left(\frac{5}{4}\right) = 2.23$ hr. The maximum concentration is $i(2.23) = 10(e^{-0.4(2.23)} - e^{-0.5(2.23)}) = 0.819$. This graph starts at $(0, 0)$ and asymptotically approaches zero for large time. A graph of the insulin concentration is below also.

d. The rate of change of insulin per hour was computed above ($i'(t)$). The rate of change at $t = 1$ is $i'(1) = 5e^{-0.5} - 4e^{-0.4} = 0.351$ units/hour.



glucose



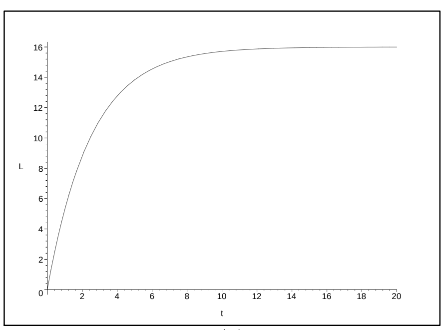
insulin

17. a. From the von Bertalanffy equation, it is easy to see that the graph passes through the origin, giving the t and L-intercepts to both be 0. As $t \to \infty$, $L(t) \to 16$, so there is a horizontal asymptote of $L = 16$. The graph of the length of the sculpin is below to the left.

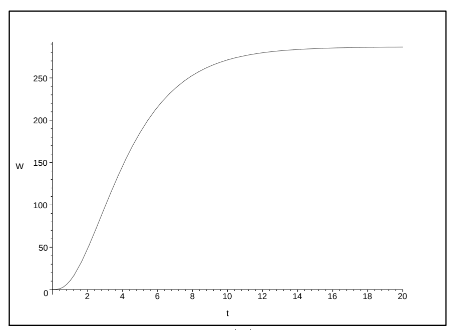
b. The composite function satisfies:

$$W(t) = 0.07\left(16(1 - e^{-0.4t})\right)^3 = 286.72(1 - e^{-0.4t})^3.$$

This function again passes through the origin, and it is easy to see that it has a horizontal asymptote at $W = 286.72$.



$L(t)$



$W(t)$

c. We apply the chain rule to differentiate $W(t)$. The result is

$$W'(t) = 3 \cdot 286.72(1 - e^{-0.4t})^2(0.4)e^{-0.4t} = 344.064(1 - e^{-0.4t})^2 e^{-0.4t}.$$

The second derivative combines the product rule and the chain rule, giving:

$$\begin{aligned} W''(t) &= 344.064\left(-0.4(1 - e^{-0.4t})^2 e^{-0.4t} + 2(1 - e^{-0.4t})0.4e^{-0.4t}e^{-0.4t}\right) \\ &= 137.6256(1 - e^{-0.4t})e^{-0.4t}\left(-(1 - e^{-0.4t}) + 2e^{-0.4t}\right) \\ &= 137.6256(1 - e^{-0.4t})e^{-0.4t}\left(3e^{-0.4t} - 1\right). \end{aligned}$$

The point of inflection is when the sculpin has its maximum weight gain, and this occurs when

$$W''(t) = 137.6256(1 - e^{-0.4t})e^{-0.4t}\left(3e^{-0.4t} - 1\right) = 0.$$

or

$$(3e^{-0.4t} - 1) = 0 \quad \text{or} \quad e^{0.4t} = 3 \quad \text{or} \quad t = \frac{5\ln(3)}{2} \simeq 2.7465.$$

The maximum weight gain is

$$W'(2.7465) = 50.97 \text{ g/yr}.$$

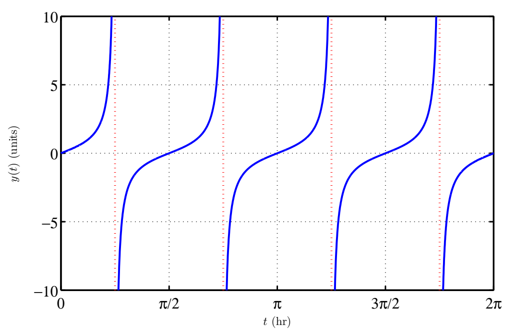
18. a. The derivative is given by

$$f'(t) = \frac{2\cos(2t)\cos(2t) + 2\sin(2t)\sin(2t)}{\cos^2(2t)} = \frac{2}{\cos^2(2t)},$$

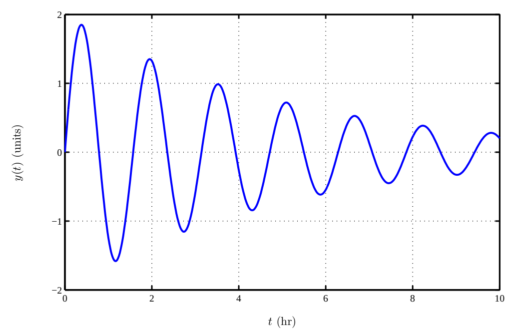
since $\sin^2(2t) + \cos^2(2t) = 1$. It follows that $f'(0) = \frac{2}{\cos^2(0)} = 2$. Notice that since the denominator is squared, it follows that the derivative is always positive for all t that the derivative is defined.

b. $f(t)$ is zero when $\sin(2t) = 0$. The sine function is zero when its argument is an integer multiple of π. For $t \in [0, 2\pi]$, $f(t) = 0$ at $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$. The cosine function is zero when its argument is $\pi/2 + n\pi$ for n an integer. Thus, the vertical asymptotes occur halfway between zeroes of f, so at $t = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

c. The graph of $f(t)$ for $t \in [0, 2\pi]$ is below to the left.



$f(t) = \tan(2x)$



Damped Spring

19. a. The damped spring-mass system, $y(t) = 2e^{-0.2t}\sin(4t)$, has $y(t_n) = 0$ when $4t_n = n\pi, \quad n = 0, 1, ...$ or $t_n = \frac{n\pi}{4}$.

b. The velocity satisfies:

$$\begin{aligned} v(t) = y'(t) &= 8e^{-0.2t}\cos(4t) - 0.4e^{-0.2t}\sin(4t) \\ &= 4e^{-0.2t}(2\cos(4t) - 0.1\sin(4t)) \end{aligned}$$

c. The absolute maximum occurs when $2\cos(4t) = 0.1\sin(4t)$ or $\tan(4t_{max}) = 20$. It follows that $t_{max} = \frac{1}{4}\arctan(20) \approx 0.3802$ sec. Thus, the maximum is

$$y(t_{max}) = 2e^{-0.2t_{max}}\sin(4t_{max}) \approx 1.8512.$$

The absolute minimum occurs at $t_{min} = t_{max} + \frac{\pi}{4} \approx 1.1656$ sec. It follows that the minimum is

$$y(t_{min}) = 2e^{-0.2t_{min}}\sin(4t_{min}) \approx -1.5821.$$

The graph is above to the right.

20. a. The basilar fiber satisfies the equation $z(t) = 20e^{-0.5t}\sin(10t)$ and vibrates through zero when the argument of $\sin(10t)$ equals $n\pi$ for n an integer. It follows that the zeroes occur when $t = \frac{n\pi}{10}, \quad n = 0, 1, ...$

b. The velocity is given by

$$\begin{aligned} v(t) = z'(t) &= 200e^{-0.5t}\cos(10t) - 10e^{-0.5t}\sin(10t) \\ &= 10e^{-0.5t}\left(20\cos(10t) - \sin(10t)\right) \end{aligned}$$

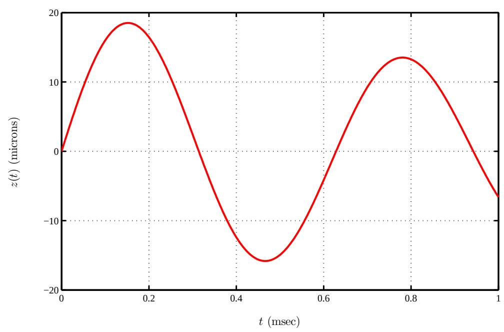
c. The absolute maximum occurs when $20\cos(10t) = \sin(10t)$, so $\tan(10t) = 20$ or $t_{max} = 0.1\arctan(20) \approx 0.1521$ msec. Thus, there is an absolute maximum at t_{max} with

$$z(t_{max}) = 20e^{-0.5t_{max}}\sin(10t_{max}) \approx 18.512\ \mu\text{m}.$$

This is followed by the absolute minimum at $t_{min} = t_{max} + \frac{\pi}{10} \approx 0.4662$ msec. with

$$z(t_{min}) = 20e^{-0.5t_{min}}\sin(10t_{min}) \approx -15.821\ \mu\text{m}.$$

The graph of $z(t)$ for $t \in [0, 1]$ is shown below.



Hair Cell fiber

21. The volume of the open box satisfies the **Objective function**

$$V(x, y) = x^2 y.$$

The **Constraint condition** on the surface area of this box is given by

$$SA = x^2 + 4xy = 600.$$

This constraint condition yields $y = \frac{600-x^2}{4x}$, which when substituted into the objective function produces a function of one variable:

$$V(x) = x^2\left(\frac{600 - x^2}{4x}\right) = \frac{1}{4}(600x - x^3).$$

Differentiating this quantity, we obtain

$$\frac{dV}{dx} = \frac{1}{4}(600 - 3x^2),$$

which when set equal to zero gives $x = 10\sqrt{2}$ cm. (Take only the positive root.) This value of x gives the optimal length of one side of the base, which when substituted into the formula above gives $y = 5\sqrt{2}$ cm. It follows that the maximum volume for this box is $V(x) = 1000\sqrt{2}$ cm^3.

22. Combining the number of drops with the energy function, we have

$$E(h) = hN(h) = h\left(1 + \frac{10}{h-1}\right) = h\left(\frac{h-1+10}{h-1}\right) = \frac{h^2+9h}{h-1}.$$

This is differentiated to give

$$E'(h) = \frac{(h-1)(2h+9) - (h^2+9h)}{(h-1)^2} = \frac{h^2-2h-9}{(h-1)^2}.$$

A minimum occurs when $h^2 - 2h - 9 = 0$, so

$$h = 1 \pm \sqrt{10} = -2.1623, 4.1623.$$

It follows that the minimum energy occurs when $h = 1 + \sqrt{10} = 4.1623$ m, which give the height that a crow should fly to minimize the energy needed to break open a walnut. At this height the average number of drops required by the crow will be:

$$N(4.1623) \approx 4.1623.$$

23. The area of the brochure is $A = xy = 125$, where x is the width of the page and y is the length of the page. The area of the printed page, which is to be maximized is given by

$$P = (x-4)(y-5).$$

From the constraint on the page area, we have $y = 125/x$, which when substituted above gives

$$P(x) = (x-4)\left(\frac{125}{x} - 5\right) = 125 - \frac{500}{x} - 5x + 20 = 145 - 500x^{-1} - 5x.$$

The maximum is found by differentiation, which gives

$$P'(x) = 500x^{-2} - 5 = \frac{5(100-x^2)}{x^2}.$$

This is zero when $x = 10$. It follows that $y = 12.5$. So the brochure has the dimensions 10×12.5 with the printed region having dimensions 6×7.5 or 45 in^2.

24. a. The time as a function of x is given by

$$T(x) = \frac{50-x}{15} + \frac{(x^2+1600)^{1/2}}{9}.$$

b. We differentiate $T(x)$ to find the minimum time,

$$T'(x) = -\frac{1}{15} + \frac{1}{9}\left(\frac{1}{2}(x^2+1600)^{-1/2}2x\right) = -\frac{1}{15} + \frac{x}{9(x^2+1600)^{1/2}}.$$

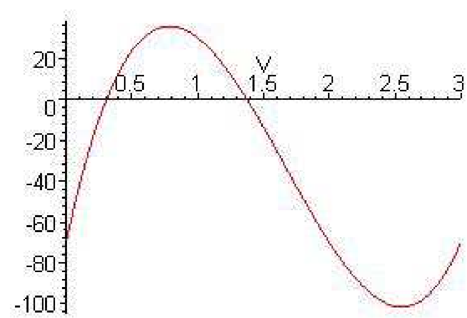
Setting this derivative equal to zero gives

$$\begin{aligned}
\frac{x}{9(x^2+1600)^{1/2}} &= \frac{1}{15} \\
5x &= 3(x^2+1600)^{1/2} \\
25x^2 &= 9(x^2+1600) \\
16x^2 &= 14400 \\
x^2 &= 900
\end{aligned}$$

This implies $x = 30$ m produces the minimum time. $T(30) = \frac{20}{15} + \frac{50}{9} = \frac{62}{9} = 6.89$ sec. We check the endpoints $T(0) = \frac{70}{9} = 7.778$ sec and $T(50) = \frac{10\sqrt{41}}{9} = 7.11$ sec, confirming the optimal escape strategy is for the rabbit to run 20 m along the road, then run straight toward the burrow.

25. a. At rest, $V(t) = -70 = 50t(t-2)(t-3) - 70$, so $50t(t-2)(t-3) = 0$. Thus, the membrane is at rest when $t = 0$, 2, and 3.

b. To find the extrema, we first write $V(t) = 50(t^3 - 5t^2 + 6t) - 70$, then the derivative is $V'(t) = 50(3t^2 - 10t + 6)$. By the quadratic formula, $t = \frac{5}{3} \pm \frac{\sqrt{7}}{3} = 0.7847, 2.5486$. Substituting these values into the membrane equation gives the peak of the action potential at $t = 0.7847$ with a membrane potential of $V(0.7847) = 35.63$ mV, while the minimum potential (most hyperpolarized state) occurs at $t = 2.5486$ with a membrane potential of $V(2.5486) = -101.56$ mV. Below is a graph for this model of membrane potential.



26. The **objective function** is given by:

$$S(x,y) = 2x^2 + 7xy.$$

The constraint condition is given by:

$$V = x^2y = 50,000 \text{ cm}^3, \quad \text{so}, \quad y = \frac{50,000}{x^2}.$$

Thus,

$$S(x) = 2x^2 + \frac{350,000}{x}.$$

Differentiating we have,

$$S'(x) = 4x - \frac{350,000}{x^2}.$$

Solving $S'(x) = 0$, so $x^3 = \frac{350,000}{4} = 87,500$ or $x = 44.395$. It follows $y = 25.37$. Thus, the minimum amount of material needed is $S(44.395) = 11,825.6$ cm^2.