1. $(0,-3)$, $(4,-1)$

equation of line: $m = \frac{-1-(-3)}{4-0} = \frac{2}{4} = \frac{1}{2}$

$y-(-3) = \frac{1}{2}(x-0)$

$y+3 = \frac{1}{2}x \longrightarrow y = \frac{1}{2}x - 3$

perpendicular slope: (negative reciprocal)

$m = -2$

equation: $y-(-1) = -2(x-4)$

$y+1 = -2x+8 \longrightarrow y = -2x+7$

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2. $43\text{ lb} \cdot \frac{1\text{ Kg}}{2.2046\text{ lb}} \approx 19.5\text{ Kg}$

$(°F - 32)\frac{5}{9} = °C \longrightarrow (102-32)\frac{5}{9} = 70 \cdot \frac{5}{9} \approx 38.9°C$

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3. $f(x) = 2x-1$

x-int: (let $y=0$)

$2x-1=0 \rightarrow x = \frac{1}{2}$

$(\frac{1}{2}, 0)$

y-int: (let $x=0$)

$2(0)-1 = -1 \rightarrow (0,-1)$

$m=2$

$g(x) = 15+2x-x^2$

x-int: $15+2x-x^2 = 0$

$-(x-5)(x+3) = 0$

$x = 5, -3$

$(5,0)$, $(-3,0)$

y-int: $15+2(0)-(0)^2 = 15$

$(0,15)$

vertex: $\left(-\frac{b}{2a}, g(-\frac{b}{2a})\right)$

$x = \frac{-2}{2(-1)} = 1$

$g(1) = 15+2(1)-(1)^2 = 16$ $(1,16)$

points of intersection:

set $f(x) = g(x)$

$2x-1 = 15+2x-x^2$

$16 - x^2 = 0$

$x = 4, -4$

$f(4) = g(4) = 7 \rightarrow (4,7)$

$f(-4) = g(-4) = -9 \rightarrow (-4,-9)$

5. Pick two points from data to find slope.
Say, $(0, 1000)$ & $(1, 940)$

$$m = \frac{940-1000}{1-0} = -60$$

$V - 1000 = -60(t-0) \longrightarrow V = 1000 - 60t$

All the water is lost when Volume $=0$

$$1000 - 60t = 0 \longrightarrow t = \frac{1000}{60} \approx 16.7 \text{ weeks}$$

6. a)

week (t)	Height (cm) (h)	Model h(t)	Square Error
0	10.5	$2(0)+11 = 11$	$(11-10.5)^2 = 0.25$
1	14	$2(1)+11 = 13$	$(13-14)^2 = 1$
2	15.5	$2(2)+11 = 15$	$(15-15.5)^2 = 0.25$
4	18	$2(4)+11 = 19$	$(19-18)^2 = 1$

given: $h(t) = 2t + 11$

$J = \text{sum}: 0.25 + 1 + 0.25 + 1 = 2.5$

b) $h(3) = 2(3)+11 = 17$ cm $h(5) = 2(5)+11 = 21$ cm

7. a) $g(P) = 0.1P\left(1 - \frac{P}{400}\right)$

population is at equilibrium when $g(P) = 0$.

$$0.1P\left(1 - \frac{P}{400}\right) = 0$$

$0.1P = 0$ $1 - \frac{P}{400} = 0$

$P = 0$ $P = 400$

b) $g(P) = 0.1P - 0.00025P^2$ (I distributed above equation)

vertex: $\left(\frac{-b}{2a}, g\left(\frac{-b}{2a}\right)\right)$

$$P = -\frac{0.1}{2(-0.00025)} = \frac{0.1}{0.0005} = 200$$

$g(200) = 10$ vertex: $(200, 10)$

10. c) $\ln c = 1.3$, $\ln d = -0.5$

$$\frac{\ln c^3 - \ln c + \ln 1}{\ln c + \ln e} = \frac{3\ln c - \ln c + 0}{\ln c \quad + 1}$$

$$= \frac{3(1.3) - 1.3}{1.3 + 1} \approx 1.13$$

d) $$\frac{\ln(c^2 d) - \ln 1}{\ln\left(\frac{c}{d}\right) - \ln e} = \frac{\ln c^2 + \ln d - 0}{\ln c - \ln d - \ln e}$$

$$= \frac{2\ln c + \ln d - 0}{\ln c - \ln d - 1} = \frac{2(1.3) + (-0.5)}{1.3 - (-0.5) - 1} \approx 2.625$$

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11. When given a rational function:

- denominator $\neq 0$
- V.A. occurs when denominator $= 0$
- H.A. : 1) degree numerator > degree denom. NO HA
  2) degree num = degree denon
  leading coefficients
  3) degree num < degree denom
  HA @ $y = 0$

d) $y = \sqrt{16 - 2x}$ no imaginary, only want real-valued solutions

Domain: $\sqrt{16 - 2x} \geq 0$

$16 - 2x \geq 0$

$16 \geq 2x$

$x \leq 8$

- exponential functions have H.A.
- logarithmic functions have V.A. where input $= 0$.

11. g) $y = 20 - 5e^{-0.5x}$

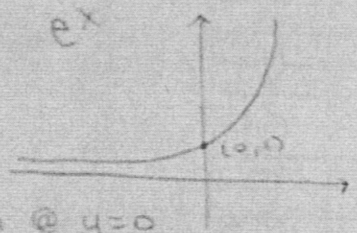
Domain: $(-\infty, \infty)$ all real #'s

think of e^x & graph (has HA @ $y=0$ when $x \to -\infty$)

$y = \underbrace{20}_{\text{causes a vertical shift of 20}} - 5e^{-0.5x}$ ← causes a reflection about y-axis

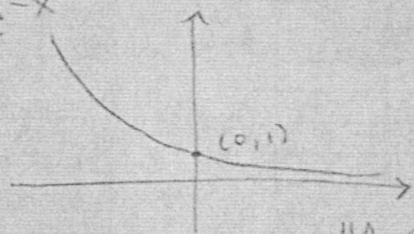
so HA @ $y = 20$ when $x \to \infty$

e^x



HA @ $y=0$ when $x \to -\infty$

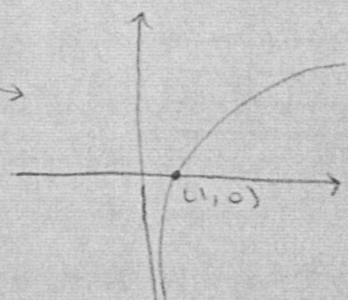
e^{-x}



HA @ $y=0$ when $x \to \infty$

h) $y = 6\ln(5-x) - 2$

think of $\ln(x)$ & graph →



VA @ $x=0$ when $y \to -\infty$

Domain: need input >0

$5 - x > 0 \to x < 5$

VA: $5 - x = 0 \to x = 5$ when $y \to -\infty$

m) $y = \dfrac{8x+5}{6-2x}$

VA: $6 - 2x = 0 \to x = 3$

HA: degree num = degree denom so take leading coefficients

$y = \dfrac{8}{-2} = -4$

12. $y(x) = 5\sin(3x) - 4$, $x \in [0, 2\pi]$

amplitude: 5 period: $\frac{2\pi}{\omega} = \frac{2\pi}{3}$

vertical shift: -4

Since $\sin(x)$ has a max @ $x = \pi/2$,

our function has a max @ $\underbrace{3x}_{\text{input (argument)}} = \pi/2 \rightarrow x = \pi/6$

all max occur at $x = \frac{\pi}{6} + \underbrace{\frac{2\pi}{3}}_{\text{period}} n$, n is any integer

* but we only want x's in $[0, 2\pi]$ *

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13. a) $y(x) = 2 - 4\cos(2x)$, $x \in [0, 2\pi]$

amplitude: 4, vertical shift: 2, period: $\frac{2\pi}{2} = \pi$

Since $\cos(x)$ has max @ $x=0$

our function has max @ $2x = 0 \rightarrow \underline{x = 0}$

add period to get next max: $x = \underline{0} + \overset{\text{period}}{\pi} = \underline{\pi}$

$x = \underline{\pi} + \underset{\text{period}}{\pi} = 2\pi$

all max : $x = 0, \pi, 2\pi$

b) equivalent model:

$$y(x) = A + B\cos(\omega(x - \phi))$$

** A phase shift of half a period creates an equivalent sine or cosine model w/ sign of amplitude reversed. **

$(\phi + \frac{1}{2}T)$

ϕ from part (a) is 0 and period (T) is π.

So, $\phi_{new} = \phi_{old} + \frac{1}{2}T$

$= 0 + \frac{1}{2}\pi = \frac{\pi}{2}$

$y(x) = 2 + 4\cos(2(x - \pi/2))$

14. a) $y(t) = 7 - 4\cos\left(\frac{\pi}{8}(t-5)\right)$, $t \in [0, 20]$

period: $\frac{2\pi}{\pi/8} = 2\pi \cdot \frac{8}{\pi} = 16$ amplitude: 4

vertical shift: 7

phase shift: 5

max of $\cos(t)$ occurs @ $t=0$

min of $\cos(t)$ occurs @ $t=\pi$

our max or min occurs when $\frac{\pi}{8}(t-5) = 0 \rightarrow t = 5$

next max or min occurs $t = 5 + \underset{\uparrow \text{period}}{16} = 21$ but $21 \notin [0, 20]$ so DON'T include.

our max or min occurs when $\frac{8}{\pi}\frac{\pi}{8}(t-5) = \pi \cdot \frac{8}{\pi}$

$t - 5 = 8 \rightarrow t = 13$

how do I know if 5 or 13 is the max/min?

$y(5) = 3$ and $y(13) = 11$

so max @ $t=13$ and min @ $t=5$

b) want: $y(t) = A + B\cos(\omega(t-\phi))$

$\phi_{new} = \phi_{old} + \frac{1}{2}T$

$= 5 + \frac{1}{2}(16) = 13$

$y(t) = 7 + 4\cos\left(\frac{\pi}{8}(t-13)\right)$

c) want: $y(t) = C + D\sin(v(t-\psi))$

** $\sin(\omega(x-\phi_1)) = \cos(\omega(x-\phi_2))$ **

$\phi_1 = \phi_2 - \frac{\pi}{2\omega}$

$\phi_1 = 13 - \frac{\pi}{2(\pi/8)} = 13 - \frac{\pi}{\pi/4} = 13 - \pi \cdot \frac{4}{\pi} = 13 - 4$
$= 9$

$y(t) = 7 + 4\sin\left(\frac{\pi}{8}(t-9)\right)$

15. a) Follow steps from 6 (a)

b) $L(H) = 1.7H + 10$

$L(81) = 1.7(81) + 10 = 148$ cm - Borzoi

Border Collie: $L = 85$

$85 = 1.7H + 10$

$75 = 1.7H \rightarrow H \approx 44$ cm

17. a)

c (mM)	A	square error
1	0.5	$[0.5 - m(1)]^2$
3	1.7	$[1.7 - m(3)]^2$
6	3.2	$[3.2 - m(6)]^2$

$J(m) = (0.5 - m)^2 + (1.7 - 3m)^2 + (3.2 - 6m)^2$

SSE

⋮ Simplify (FOIL & combine like terms)

$J(m) = 46m^2 - 49.6m + 13.38$

b) $A = 22$

$m = 0.5391 \rightarrow A = 0.5391c$

$22 = 0.5391c \rightarrow c = 4.08$ mM

19. a) $(\underset{W}{560}, \underset{F}{390})$ $(\underset{W}{2520}, \underset{F}{1210})$

find slope m, and intercept b.

b)

Weight	Feed
560	390
2520	1210

ln(Weight)	ln(Feed)
6.328	5.966
7.832	7.098

$F = kW^a$

$\ln F = \ln(kW^a)$

$\ln F = a \ln W + \ln k$

$a = \frac{(5.966 - 7.098)}{(6.328 - 7.832)} \approx .753$

pick point to find intercept, ln k

19. b) continued

$5.966 = .753(6.328) + \ln k$

$\ln k = 5.966 - .753(6.328)$

≈ 1.201

$$e^{\ln k} = e^{1.201} \rightarrow k = e^{1.201} \approx 3.33$$

So, $F = 3.33 W^{.753}$

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24. a) $H_{n+1} = 1.02 H_n$

$H_0 = 2000$

$H_1 = 1.02(2000) = 2040$

$H_2 = 1.02(2040) = 2080.8$

General Solution:

$n = 0: \quad H_1 = 1.02 H_0$

$n = 1: \quad H_2 = 1.02 H_1 = 1.02(1.02 H_0) = 1.02^2 H_0$

Notice pattern: $H_n = 1.02^n H_0$

$H_n = 2000(1.02)^n$

b) $G_{n+1} = 1.03 G_n$

$G_n = 200(1.03)^n$

Determine how long it takes for this population to double:

set equal to twice initial

$(2)200 = 200(1.03)^n$

$400 = 200(1.03)^n$

$2 = 1.03^n$

$\ln 2 = n \ln 1.03$

$n = \dfrac{\ln 2}{\ln 1.03}$

$n \approx 23.5$ generations

25. a) 1960 179.3 million $n=0$

1980 226.5 $n=20$

General solution : $P_n = (1+r)^n P_0$

$P_0 = 179.3$

$P_{20} = (1+r)^{20} \cdot 179.3 = 226.5$

$(1+r)^{20} = 1.2632$

$1+r = 1.2632^{1/20} = 1.01175$

$r = 0.011753$

* can't use growth rate formula from lab because these are NOT consecutive years.

$P_n = 179.3(0.011753)^n$

b) 2000 corresponds to $n=40$

$P_{40} = 179.3(0.011753)^{40} \approx 286.1$ million

$$\frac{286.1 - 281.4}{281.4} * 100 \approx 1.7\%$$