1. a. Consider a model with immigration given by

$$p_{n+1} = 0.8p_n + 300,$$

with an initial population of $p_0 = 500$. Determine the populations at the next three time intervals, p_1, p_2, and p_3.

b. Find all equilibria and determine the stability of these equilibria.

2. A man with a chronic lung problem has a tidal volume, V_i, of 300 ml. For this experiment, Helium, He, is used to determine the functional reserve capacity, V_r. (Note that $V_r = (1-q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1-q)c_n + q\gamma,$$

where $\gamma = 5.2$ ppm.

a. The man is given an enriched mixture of air to breathe that contains 50 ppm of He. Experimentally, the concentration of He in the first two measured breaths after breathing the enriched mixture are given by $c_0 = 50$ and $c_1 = 44.6$ ppm. Use c_0 and c_1 to find q, then determine the functional reserve capacity, V_r.

b. Use your model to find the expected concentration of Helium in this patient's 3^{rd} breath, c_3. What is the equilibrium concentration of Helium in the patient's lungs? What is the stability of this equilibrium concentration?

3. Below are data on the population of insect pests living in a survey area. The insect reproduces according to a Malthusian growth model and disperses (emigrates) to surrounding regions at a constant rate. The population model for this insect pest is given by

$$P_{n+1} = (1+r)P_n - \mu,$$

where r is the rate of growth (per week) and μ is the number of pests dispersing each week to surrounding regions.

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of the insect pests for weeks 3 and 4.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Week	Insects
0	500
1	630
2	825

4. A woman with a chronic lung problem breathes a supply of air enriched with Helium. Experimental measurements show the following concentrations of the exhaled air after she resumes normal breathing in the room.

Breath Number	0	1	2
Conc. of He (ppm)	400	352	310

The concentration of Helium in the room, γ, is not known, but assumed to be constant.

a. Assume a breathing model of the form:

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

Use the data above to find the constants q, the fraction of air exchanged, and γ, the ambient concentration of Helium. Then determine the concentration of Helium in the next two breaths, c_3 and c_4.

b. Find the equilibrium concentration of Helium in the subject's lungs based on this breathing model. What is the stability of this equilibrium concentration? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $c_{n+1} = c_n$, showing all intercepts ($c_n \geq 0$) and points of intersection.

5. a. An ecological survey for a particular invasive species of fish is conducted at a lake. This species reproduces in the lake and is predated upon by other fish in the lake. There is also a constant immigration of the fish from a river feeding the lake. The survey finds that the population of this fish satisfies the discrete linear growth model

$$F_{n+1} = 0.86\,F_n + 280, \qquad F_0 = 100,$$

where F_n is the population of fish in the n^{th} year and $F_0 = 100$ is the initial population surveyed. Find the populations F_1 and F_2. Find the equilibrium population, F_e. Is this a stable or unstable population?

b. The discrete model above is approximated very well by the continuous model

$$F(t) = 2000 - 1900\,e^{-0.15t}.$$

Find the F-intercept and any horizonal asymptotes for $F(t)$, then sketch a graph of the function.

c. Use $F(t)$ to find the slope of the secant line between $t = 5$ and $t = 6$. Also, find the slope of the secant line between $t = 5$ and $t = 5.1$.

6. A ball, which is thrown vertically upward with a velocity of 48 ft/sec and falls under the influence of gravity without air resistance from a 160 ft cliff, satisfies the equation

$$h(t) = 160 + 48t - 16t^2,$$

where h is in feet and t is in seconds.

a. Find the average velocity of the ball between the times $t = 0$ and $t = 2$. Also, find the average velocity between the times $t = 1$ and $t = 1.2$ and between the times $t = 1$ and $t = 1.01$

b. Sketch a graph of $h(t)$, showing crucial points, including the h-intercept, the maximum height, and when the ball hits the ground. Approximate the velocity with which the ball hits the ground by finding the average velocity of the ball between the time the ball hits the ground and 0.001 sec before it hits the ground.

7. Fish continue to grow throughout their life, but their growth slows with age. The leopard shark (*Triakis semifiate*) is a common ovoviviparous, benthic shark in the San Diego waters. A good approximation to the growth of the leopard shark uses the von Bertalanffy equation for fish growth and is given by

$$L(t) = 2.1 - 1.9e^{-0.25t},$$

where L is in meters and t is in years.

a. Determine any asymptotes. Find the approximate length of a leopard shark at birth and ages 1, 5, and 10 years. Sketch a graph of the length of a leopard shark. What is the maximum length of a leopard shark and at what age does it reach 90% of that length?

b. Find the average rate of growth between the ages of 1 and 5 years, between the ages of 5 and 10 years, between the ages of 5 and 6 years, and between the ages of 5 and 5.01 years. Which of these gives the best approximation to the derivative (instanteous rate of growth) at $t = 5$ years?

8. a. A serval is a 20-40 pound wild cat that has an incredible ability to leap and catch birds. Its predation success rate ranks among the highest of any animals. Suppose that a serval is sitting on a ledge 16 ft above the ground and can jump with a vertical velocity of 24 ft/sec. If we ignore air resistance and use an acceleration from gravity of -32 ft/sec^2, then the height of the serval above the ground, $h(t)$, is given by the formula

$$h(t) = 16 + 24t - 16t^2.$$

Find the range of heights for a bird flying above the serval that allows this predator a chance to catch it.

b. Find the average velocity of the serval for the intervals $t \in [0, \frac{1}{4}]$, $t \in [\frac{1}{2}, 1]$, and $t \in [1, \frac{5}{4}]$.

c. The velocity, $v(t)$, satisfies $v(t) = h'(t)$. Find $v(t)$ and compute $v(1)$.

d. Determine the time when the serval hits the ground and the velocity with which it hits the ground. Sketch a graph of the height of the serval as a function of t.

9. An impala is migrating across a field that has been fenced with a 180 cm fence. To escape it needs to jump this fence. Assume that the impala jumps the fence with just enough vertical velocity, v_0 to clear it. If the height (in cm) of the impala is given by

$$h(t) = v_0 t - 490t^2.$$

a. Use the height of the fence (maximum height) and the function describing the height of the impala, $h(t)$, to determine the vertical velocity, v_0, then determine how long the impala is in the air.

b. With this value of v_0, find the average velocity of the impala between $t = 0$ and $t = 0.5$.

10. a. Find the slope of the secant line through the points $(2, f(2))$ and $(2+h, f(2+h))$ for the function

$$f(x) = \frac{x-2}{2x+2}.$$

b. Continuing from Part a, let h get small and determine the slope of the tangent line through $(2, f(2))$, which gives the value of the derivative of $f(x)$ at $x = 2$. Give the equation of this tangent line.

c. Sketch a graph of $f(x)$ giving x and y-intercepts, vertical and horizontal asymptotes, and including the tangent line at $(2, f(2))$.

11. Differentiate the following: (Do **NOT** simplify!)

a. $f(x) = 6x^3 + \dfrac{2}{x^2} - e^{2x}(x^2 - 9)$,

b. $g(x) = 2e^{-3x} + \ln(x^2) - 5$,

c. $h(x) = 2x^6 \ln(x) - e^{\sin(2x)} + \dfrac{e^{-4x}}{2}$,

d. $b(x) = \ln(\cos(3x)) - e^{x^2+4x}$,

e. $q(x) = \dfrac{2 + e^{2x}}{x^2 - 3} - (x^2 - \sin^3(x^2))^4$,

f. $k(t) = \dfrac{1}{4}t^2 - \dfrac{4}{\cos(t^2+2)} + \dfrac{4}{\sqrt{t}}$,

g. $r(x) = e^{2x}(x^3 - 5x + 7)^4 - e^{-x}\cos(2x)$,

h. $w(x) = \dfrac{x^4 + e^{-2x}}{x^3 + \cos(4x)} + \dfrac{7x}{\sqrt{x^2 + 2x + 5}}$.

12. For each of the following functions, give the domain. Find all x and y-intercepts and any asymptotes, if they exist. Find the derivative of the functions, then determine any maxima or minima. Give both the x and y values. (Optional for more practice differentiating: Write the second derivative and find any points of inflection.) Sketch the graph of the function.

a. $y = 27x - x^3$,

b. $y = 18x^2 - x^4$,

c. $y = 4xe^{-0.02x}$,

d. $y = (x+3)\ln(x+3)$,

e. $y = (x-4)e^{2x}$,

f. $y = \dfrac{10(x-2)}{(1+0.5x)^3}$,

g. $y = x + \dfrac{4}{x}$,

h. $y = \dfrac{4x^2}{x+3}$

13. Body temperatures of animals undergo circadian rhythms. A subject's temperature is measured from 8 AM until midnight, and his body temperature, T (in °C), is best approximated by the cubic polynomial

$$T(t) = 0.002(t^3 - 45t^2 + 600t + 16000),$$

where t is in hours.

a. Find the rate of change in body temperature $\frac{dT}{dt}$. What is the rate of change in body temperature at noon $t = 12$?

b. Use the derivative to find when the maximum temperature of the subject occurs and when the minimum temperature of the subject occurs. What are the body temperatures at those times? State the intervals of time where the subject's body temperature is decreasing.

14. The distribution of seeds from around a plant satisfies an exponential distribution. It is more likely to find seeds close to a plant than it is far away. Suppose that experimental measurements fitting the seed distribution radially from the plant satisfies

$$P(r) = 0.04r\, e^{-0.2r},$$

where P is the probability of finding a seed r meters from the plant. Find the distance r and the probability $P(r)$ at which a seed is most likely to land. That is find the maximum probability from the function above. Sketch a graph of $P(r)$ showing any intercepts, asymptotes, and local extrema.

15. A tumor growing according to Gompertz's model satisfies the growth law

$$G(N) = N(0.8 - 0.04\ln(N)) \text{ (cells/day)},$$

where N is the number of tumor cells and the time units are days.

a. Find the equilibrium number of tumor cells by solving when $G(N) = 0$.

b. Compute $G'(N)$ and determine the population at which the maximum rate of growth of the tumor is occurring. What is the maximum growth rate (in cells/day)?

c. Find the value for the growth function for $N = 2 \times 10^8$. Also, find the rate of the growth function of the tumor for this size. Describe whether the tumor is growing or decreasing according to your results, and whether the velocity of this growth/decrease is increasing or decreasing.

16. a. After ingestion of a sugar-rich meal, the concentration of glucose rises very rapidly in the blood, then through insulin, the glucose is converted to glycogen for later use as energy. An approximation for the concentration of glucose in the body after this meal is given by

$$g(t) = 80 + 150e^{-0.8t}\sin(t),$$

where t is in hours and g is in mg/100 ml of blood. Find how long it takes for the concentration of glucose in the blood to reach 80 mg/100 ml of blood for the first time after $t = 0$.

b. Find the rate of change of glucose per hour ($\frac{dg}{dt}$) for any time t. Evaluate the rate of change at $t = 1$. Determine the times, $t \geq 0$, when the blood glucose concentration is at its absolute maximum and absolute minimum, and find the concentration at those times. Sketch a graph for the concentration of glucose in the blood for $t \in [0, 6]$.

c. The release of insulin responds to this rise in glucose. Suppose that the level of insulin satisfies the function

$$i(t) = 10(e^{-0.4t} - e^{-0.5t}).$$

Find when the insulin reaches its maximum concentration and determine what its maximum concentration is. Sketch a graph of the insulin concentration.

d. Find the rate of change of insulin per hour ($\frac{di}{dt}$) for any time t, then evaluate this rate of change at $t = 1$.

17. The growth in length of sculpin is approximated by the von Bertalanffy equation

$$L(t) = 16(1 - e^{-0.4t}),$$

where t is in years and L is in cm. An allometric measurement of sculpin shows that their weight can be approximated by the model

$$W(L) = 0.07L^3,$$

where W is in g.

a. Find the intercepts and any asymptotes for the length of a sculpin, then sketch of graph showing the length of a sculpin as it ages.

b. Create a composite function to give the weight of the sculpin as a function of its age, $W(t)$. Find the intercepts and any asymptotes for $W(t)$, then sketch of graph showing the weight of a sculpin as it ages.

c. Find the derivative of $W(t)$ using the chain rule. Also, compute the second derivative, then determine when this second derivative is zero. From this information, find at what age the sculpin are increasing their weight the most and determine what that weight gain is. Be sure to give the units of weight gain.

18. Consider the function:

$$f(t) = \frac{\sin(2t)}{\cos(2t)}.$$

a. Find the derivative of $f(t)$. Evaluate $f'(0)$. Is the derivative positive, negative, or both for $t \geq 0$?

b. For $t \in [0, 2\pi]$, find all zeroes of $f(t)$. Also, determine where the function is undefined (zeroes of the denominator). The zeroes in the denominator give the location of the vertical asymptotes.

c. Use the information from Parts a and b to sketch a graph of $f(t)$ for $t \in [0, 2\pi]$.

19. A damped spring-mass system has a solution of the form

$$y(t) = 2e^{-0.2t}\sin(4t),$$

where $y(t)$ measures the distance in centimeters from the equilibrium position and t is in seconds.

a. Determine the times when $y(t) = 0$.

b. Find the velocity of the mass by computing the derivative, $v(t) = y'(t)$.

c. Find the times $t \geq 0$, when the mass is at its absolute maximum and absolute minimum. Also, give the maximum and minimum displacements at those times. Sketch a graph for the position of this mass.

20. The displacement of specific fibers on the basilar membrane stimulates the hair cells, which send a signal to the auditory part of the brain, indicating a particular wavelength of sound has been heard. For a given tone, assume that the basilar fiber vibrates according to the equation:

$$z(t) = 20e^{-0.5t}\sin(10t),$$

where $z(t)$ measures the distance in microns from the rest position of the fiber and t is in milliseconds.

a. Determine the times when $z(t) = 0$ for $t \geq 0$.

b. Find the velocity of the basilar fiber by computing the derivative, $v(t) = z'(t)$.

c. Find the times $t \geq 0$, when $z(t)$ is at its absolute maximum and absolute minimum. Also, give the maximum and minimum displacements at those times. Sketch a graph for the position of the basilar fiber. Sketch a graph of $z(t)$ for $t \in [0, 1]$.

21. An open box with a square base is to be constructed with 600 cm^2 of material. Find its dimensions that maximize the volume.

22. Suppose that a study finds that the best fit for the number of drops required to break open a walnut satisfies the equation

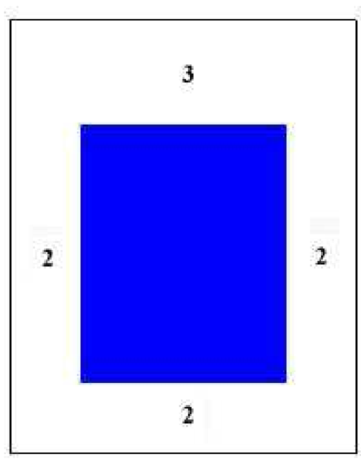
$$N(h) = 1 + \frac{10}{h-1},$$

where h is the height of the drop in meters. The energy used to break open a walnut is given by the function

$$E(h) = hN(h).$$

Find the height that a crow should fly to minimize the energy needed to break open a walnut. What is the average number of drops that the crow will need to break the walnut from this height?

23. A brochure is to have an area of 125 in^2, with a 3 in margin at the top and 2 in margins on the sides and bottom. Find the dimensions of the brochure that allow the maximum printing area. (See figure below.)

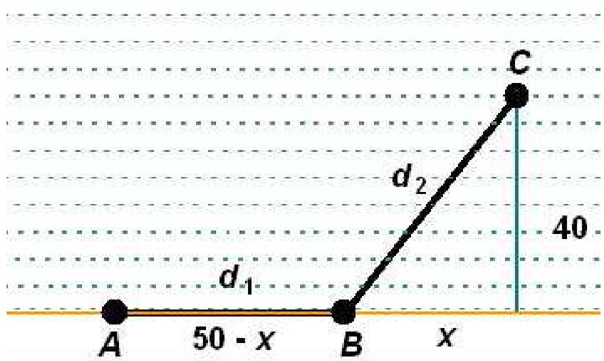


24. A rabbit is being chased by a predator and wants to reach its burrow as quickly as possible. (See the diagram below.) Assume that it is initially at Point A with the burrow residing at Point C. Assume it can run along the road at $v_1 = 15$ m/sec and through the brush at $v_2 = 9$ m/sec. The burrow is 40 m from the road, and the rabbit is 50 m up the road from the closest point of the burrow to the road. If the distance it runs along the road (from A to B) is d_1 and the distance it runs in the brush (from B to C) is d_2, then the time for the rabbit to reach the burrow is given by the formula

$$T = \frac{d_1}{v_1} + \frac{d_2}{v_2}.$$

a. Use the diagram below to form an expression for the time as a function of x (the distance down the road to where the rabbit enters the brush), $T(x)$.

b. Use your expression for the time $T(x)$ to find the minimum time for the rabbit to reach its burrow. Give both the distance x and the time at the minimum.



25. A typical nerve action potential is characterized by a sharp outflowing of sodium ions leading to a depolarization of the nerve membrane. Next potassium ions flow inward causing a repolarization. There is a period of time afterwards that the membrane is hyperpolarized, which prevents further stimulation of the nerve before returning to resting potential. A cubic equation that approximates the data for a typical action potential of a nerve cell is given by

$$V(t) = 50t(t-2)(t-3) - 70,$$

where t is the time in msec following stimulation of the nerve cell and V is membrane potential in mV.

a. The membrane potential is considered at rest when $V(t) = -70$ mV. Find the times when this model predicts the membrane is at rest.

b. Find the time and potential of the membrane at the peak of the action potential. Also, find the time and membrane potential of the membrane when it is most hyperpolarized (minimum potential). Sketch a graph of this function for $0 \le t \le 3$.

26. A student in ecology needs to design two experimental holding pens for their species of animal. Below is a design of the two pens that are each square on their ends. The animal needs 50,000 cm^3 of space in each of these holding pens. (Note that the interior side is shared by both pens, so be **very** careful when you count the sides.) Special material is used for the sides of the pens and back, while the front end screened with a door (so is not included in the calculations). Find the dimensions width and depth (x and y) for each of the holding pens that minimizes the surface area of the enclosed space.