1. Differentiation gives:

a. Consider

$$f_1(x) = x^3 + 8\,e^{-2x}.$$

The derivative uses the power and exponential rules, so

$$f_1'(x) = 3\,x^2 + 8(-2\,e^{-2x}) = 3\,x^2 - 16\,e^{-2x}.$$

b. Consider

$$f_2(x) = 4\,\sin(2x+1).$$

The derivative uses rules for differentiation of the sine function, so

$$f_2'(x) = 4(2\,\cos(2x+1)) = 8\,\cos(2x+1).$$

c. Consider

$$f_3(x) = \frac{1}{4x^2} + 7 = \frac{1}{4}x^{-2} + 7.$$

The derivative uses the power rule, so

$$f_3'(x) = \frac{-2}{4}x^{-3} + 0 = -\frac{1}{2x^3}.$$

d. Consider

$$f_4(x) = 2\,\ln(x^2+5).$$

The derivative uses rules for differentiation of the logarithm function, so

$$f_4'(x) = 2\left(\frac{2x}{x^2+5}\right) = \frac{4x}{x^2+5}.$$

e. Consider

$$f_5(x) = x^2\cos(3x).$$

The derivative uses the product rule, so

$$f_5'(x) = x^2(-3\,\sin(3x)) + (2x)\cos(3x) = -3\,x^2\sin(3x) + 2x\,\cos(3x).$$

f. Consider

$$f_6(x) = 4\,e^{x^3+1}.$$

The derivative uses the chain rule, so

$$f_6'(x) = 4\,e^{x^3+1}(3\,x^2) = 12\,x^2e^{x^3+1}.$$

g. Consider

$$f_7(x) = 2x\,e^{x/2}.$$

The derivative uses the product rule, so

$$f_7'(x) = 2\left(x\left(\frac{1}{2}\,e^{x/2}\right) + 1\cdot e^{x/2}\right) = x\,e^{x/2} + 2\,e^{x/2}.$$

h. Consider

$$f_8(x) = \sqrt{x^4+6} = (x^4+6)^{\frac{1}{2}}.$$

The derivative uses the chain rule, so

$$f_8'(x) = \frac{1}{2}(x^4+6)^{-\frac{1}{2}}(4\,x^3) = \frac{2\,x^3}{\sqrt{x^4+6}}.$$

i. Consider

$$f_9(x) = 3\,\cos(2e^x - 1).$$

The derivative uses the chain rule, so

$$f_9'(x) = -3\,\sin(2e^x-1)(2e^x) = -6\,e^x\sin(2e^x-1).$$

j. Consider

$$f_{10}(x) = 3\sqrt{x} + \ln(4x) = 3\,x^{1/2} + \ln(4) + \ln(x).$$

The derivative uses the chain rule, so

$$f_{10}'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{x}.$$

2. a. Consider

$$y(x) = 2 + 7\left(e^{-0.01x} - e^{-0.5x}\right).$$

The y-intercept is $y(0) = 2 + 7(1-1) = 2$. To find the x-intercept, we solve

$$0 = 2 + 7\left(e^{-0.01x} - e^{-0.5x}\right) \qquad \text{or} \qquad 7\left(e^{-0.01x} - e^{-0.5x}\right) = -2.$$

This is a transcendental equation, so there is no direct way to solve for x. This can be solved in Maple to give the x-intercept, $x = -0.5105756$. (Alternately, we can evaluate $y(0.5)$ and $y(0.52)$ to see that the function changes signs and crosses the x-axis.) There is a horizontal asymptote for $x \to +\infty$ with $y = 2$.

Next we find the derivative:

$$y'(x) = 7\left(-0.01\,e^{-0.01x} + 0.5\,e^{-0.5x}\right).$$

The critical points are found by setting $y'(x) = 0$. It follows that

$$0.01\,e^{-0.01x} = 0.5\,e^{-0.5x} \qquad \text{or} \qquad \frac{e^{-0.01x}}{e^{-0.5x}} = \frac{0.5}{0.01}.$$

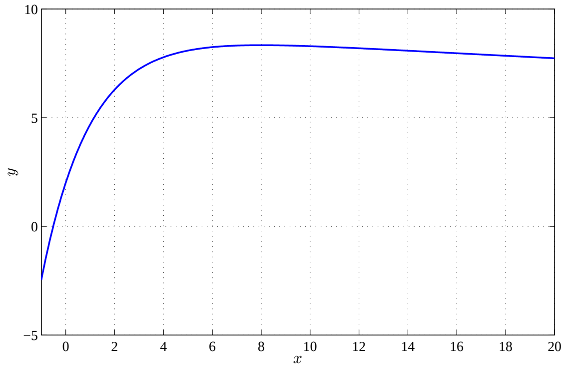
so

$$e^{0.49x} = 50 \quad \text{or} \quad 0.49\,x_c = \ln(50) \quad \text{or} \quad x_c = \frac{\ln(50)}{0.49} = 7.9837.$$

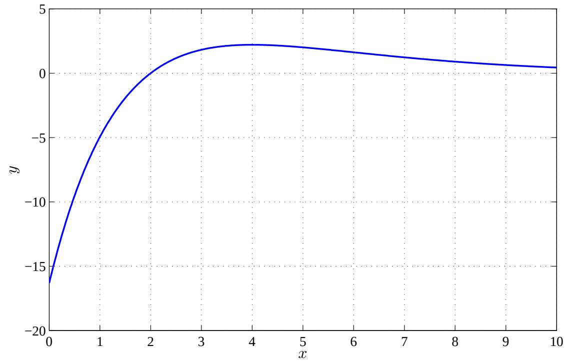
Substituting this value into the function, we find:

$$y(7.9837) = 2 + 7\left(e^{-0.079837} - e^{-3.9919}\right) = 8.3336,$$

which gives a maximum at $(7.9837, 8.3336)$. Below we see a graph of this function.



Problem 2a



Problem 2b

b. Consider
$$y = 3(x-2)e^{-(x-2)/2}.$$
The y-intercept is $y(0) = -6\,e^1 = -16.310$. To find the x-intercept, we solve
$$3(x-2)e^{-(x-2)/2} \qquad \text{or} \qquad x - 2 = 0 \qquad \text{or} \qquad x = 2,$$
since the exponential function is never zero. There is a horizontal asymptote for $x \to +\infty$ with $y = 0$. This uses the fact that the decaying exponential dominates the linear factor $x - 2$.

Next we find the derivative using the product rule:
$$\begin{aligned} y'(x) &= 3\left((x-2)\left(-\frac{1}{2}e^{-(x-2)/2}\right) + 1\cdot e^{-(x-2)/2}\right) \\ y'(x) &= 3\,e^{-(x-2)/2}\left(2-\frac{x}{2}\right). \end{aligned}$$
The critical points are found by setting $y'(x_c) = 0$. Since the exponential function is never zero, it follows that
$$\left(2 - \frac{x_c}{2}\right) = 0 \qquad \text{or} \qquad x_c = 4.$$
Substituting this value into the function, we find:
$$y(4) = 3(4-2)e^{-(4-2)/2} = 6\,e^{-1} = 2.2073,$$
which gives a maximum at $(4, 2.2073)$. The graph of this function is above.