I-Clicker Question

Which of the following optimization **Homework** or Review problems would you most like to see worked in Lecture?

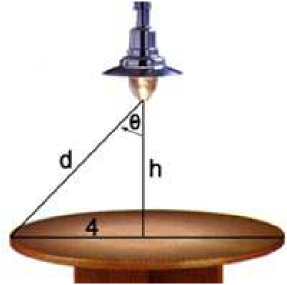
A. **Homework 3 or 4:** Optimal dimensions of an open box.

B. **Homework 11 or Review 24:** Optimal time of escape (otter or rabbit).

C. **Homework 12:** Optimal lamp illumination.

D. **Review 23:** Optimal size of brochure.

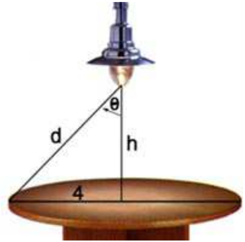
E. **Review 26:** Optimal size of holding pens.



Optimization problem seeks to find the maximum illumination by changing h if

$$I = \frac{3.3\cos(\theta)}{d^2}$$

Given the diagram for the table, find d.



A. $d = \frac{4}{\tan(\theta)}$

B. $d = 4\tan(\theta)$

C. $d = 4\sin(\theta)$

D. $d = \frac{4}{\sin(\theta)}$

E. $d = \frac{4}{\cos(\theta)}$

Given that

$$I = \frac{3.3\cos(\theta)}{d^2} \qquad \text{and} \qquad d = \frac{4}{\sin(\theta)}$$

It follows that

$$I = \frac{3.3}{16}\cos(\theta)\sin^2(\theta)$$

Given

$$I(\theta) = \frac{3.3}{16}\cos(\theta)\sin^2(\theta)$$

Find the derivative of $I(\theta)$?

A. $I'(\theta) = \frac{3.3}{16}\sin(\theta)(\cos^2(\theta) - \sin^2(\theta))$

B. $I'(\theta) = -\frac{6.6}{16}\sin^2(\theta)\cos(\theta)$

C. $I'(\theta) = \frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1)$

D. $I'(\theta) = \frac{3.3}{16}\sin(\theta)(\sin^2(\theta) - 2\cos^2(\theta))$

E. $I'(\theta) = \frac{3.3}{16}\cos(\theta)(2\cos^2(\theta) - \sin^2(\theta))$

Hint: You may need to use the identity $\cos^2(\theta) + \sin^2(\theta) = 1$

For

$$I = \frac{3.3\cos(\theta)}{d^2}$$

We have

$$I'(\theta) = \frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1)$$

For

$$I = \frac{3.3\cos(\theta)}{d^2}$$

We have

$$I'(\theta) = \frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1)$$

The optimal solution satisfies

$$\frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1) = 0$$

For

$$I = \frac{3.3\cos(\theta)}{d^2}$$

We have

$$I'(\theta) = \frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1)$$

The optimal solution satisfies

$$\frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1) = 0$$

It follows that either

$$\sin(\theta) = 0 \qquad \text{or} \qquad 3\cos^2(\theta) - 1$$

For

$$I = \frac{3.3\cos(\theta)}{d^2}$$

We have

$$I'(\theta) = \frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1)$$

The optimal solution satisfies

$$\frac{3.3}{16}\sin(\theta)(3\cos^2(\theta) - 1) = 0$$

It follows that either

$$\sin(\theta) = 0 \qquad \text{or} \qquad 3\cos^2(\theta) - 1$$

Since $\sin(\theta) = 0$ implies $\theta = 0$, which is not optimal, it follows that

$$\cos(\theta) = \frac{1}{\sqrt{3}}$$

Given

$$\cos(\theta) = \frac{1}{\sqrt{3}} = \frac{h}{d}$$

we need $\sin(\theta)$. **What is** $\sin(\theta)$**?**

A. $\sin(\theta) = \frac{2}{\sqrt{3}}$

B. $\sin(\theta) = \frac{2}{3}$

C. $\sin(\theta) = \frac{\sqrt{3}}{2}$

D. $\sin(\theta) = \frac{1}{2}$

E. $\sin(\theta) = \sqrt{\frac{2}{3}}$

Lamp Problem

We combine our results.

- The optimal solution has

$$\cos(\theta_{opt}) = \frac{1}{\sqrt{3}} = \frac{h}{d}$$

We combine our results.

- The optimal solution has

$$\cos(\theta_{opt}) = \frac{1}{\sqrt{3}} = \frac{h}{d}$$

- The hypotenuse is

$$d = \frac{4}{\sin(\theta_{opt})} \quad \text{with} \quad \sin(\theta_{opt}) = \sqrt{\frac{2}{3}}$$

We combine our results.

- The optimal solution has

$$\cos(\theta_{opt}) = \frac{1}{\sqrt{3}} = \frac{h}{d}$$

- The hypotenuse is

$$d = \frac{4}{\sin(\theta_{opt})} \quad \text{with} \quad \sin(\theta_{opt}) = \sqrt{\frac{2}{3}}$$

- It follows that

$$h = \frac{4}{\sqrt{2}}$$

Box Problem – I-Clicker

An open box with its base having a length twice its width is to be constructed with 800 square cm of material. **Find its dimensions that maximize the volume.**

Box Problem – I-Clicker

An open box with its base having a length twice its width is to be constructed with 800 square cm of material. **Find its dimensions that maximize the volume.**

Let its width be denoted x and its height be denoted y, then the volume, $V(x, y)$, of this open box satisfies:

A. $V(x, y) = 2x^2 + 6xy$

B. $V(x, y) = 2x^2 y$

C. $V(x, y) = 2xy^2$

D. $V(x, y) = x^2 y$

E. $V(x, y) = x^2 + 4xy$

An open box with its base having a length twice its width is to be constructed with 800 square cm of material. **Find its dimensions that maximize the volume.**

Let its width be denoted x and its height be denoted y, then the volume, $V(x,y)$, of this open box satisfies:

A. $V(x,y) = 2x^2 + 6xy$

B. $V(x,y) = 2x^2 y$

C. $V(x,y) = 2xy^2$

D. $V(x,y) = x^2 y$

E. $V(x,y) = x^2 + 4xy$

This is the Objective function.

The open box is to be constructed with 800 square cm of material. **Find an equation for the surface area,** $S(x, y)$**.**

The open box is to be constructed with 800 square cm of material. **Find an equation for the surface area, $S(x,y)$.**

A. $S(x,y) = 2x^2 + 4xy = 800$

B. $S(x,y) = 2x^2y + 2x^2 = 800$

C. $S(x,y) = 2xy^2 = 800$

D. $S(x,y) = 2x^2 + 6xy = 800$

E. $S(x,y) = x^2 + 4xy = 800$

The open box is to be constructed with 800 square cm of material. **Find an equation for the surface area, $S(x,y)$.**

A. $S(x,y) = 2x^2 + 4xy = 800$

B. $S(x,y) = 2x^2y + 2x^2 = 800$

C. $S(x,y) = 2xy^2 = 800$

D. $S(x,y) = 2x^2 + 6xy = 800$

E. $S(x,y) = x^2 + 4xy = 800$

This is the Constraint condition.

Objective function is

$$V(x, y) = 2x^2 y$$

with **Constraint condition**

$$S(x, y) = 2x^2 + 6xy = 800$$

Objective function is

$$V(x, y) = 2x^2 y$$

with **Constraint condition**

$$S(x, y) = 2x^2 + 6xy = 800$$

Thus, $6xy = 800 - 2x^2$, or

Box Problem

Objective function is

$$V(x,y) = 2x^2y$$

with **Constraint condition**

$$S(x,y) = 2x^2 + 6xy = 800$$

Thus, $6xy = 800 - 2x^2$, or

$$y = \frac{400}{3x} - \frac{x}{3}$$

Objective function is

$$V(x,y) = 2x^2y$$

with **Constraint condition**

$$S(x,y) = 2x^2 + 6xy = 800$$

Thus, $6xy = 800 - 2x^2$, or

$$y = \frac{400}{3x} - \frac{x}{3}$$

The objective function becomes

$$V(x) = 2x^2\left(\frac{400}{3x} - \frac{x}{3}\right) = \frac{2}{3}(400x - x^3)$$

Box Problem

Since the objective function is

$$V(x) = \frac{2}{3}(400x - x^3),$$

we differentiate to obtain

Box Problem

Since the objective function is

$$V(x) = \frac{2}{3}(400x - x^3),$$

we differentiate to obtain

$$V'(x) = \frac{2}{3}(400 - 3x^2)$$

Box Problem

Since the objective function is

$$V(x) = \frac{2}{3}(400x - x^3),$$

we differentiate to obtain

$$V'(x) = \frac{2}{3}(400 - 3x^2) = 0$$

for optimal solution.

Box Problem

Since the objective function is

$$V(x) = \frac{2}{3}(400x - x^3),$$

we differentiate to obtain

$$V'(x) = \frac{2}{3}(400 - 3x^2) = 0$$

for optimal solution.

It follows that

$$x_{opt} = \frac{20}{\sqrt{3}}$$

Since the objective function is

$$V(x) = \frac{2}{3}(400x - x^3),$$

we differentiate to obtain

$$V'(x) = \frac{2}{3}(400 - 3x^2) = 0$$

for optimal solution.

It follows that

$$x_{opt} = \frac{20}{\sqrt{3}}$$

The length, height, and volume are easily obtained from this.