1. Use the analysis of the Lotka-Volterra or Predator-Prey model to explain why the application of DDT to control scale insects in the citrus industry led to excessive application of the pesticide, which in turn resulted in many environment problems highlighted in the classic book that really started the environmental movement, Rachel Carlson's *Silent Spring* (referring to the devastation of the bird populations). The citrus groves were rapidly invaded by scale insects, which caused tremendous destruction until lady bugs, the scale insect's natural predator, were imported as a control. Give at least two reasons why use of pesticides result in escalating use of pesticides that further put farmers in debt and only enrich the chemical industry. Use modeling methods to show some smarter way to control an agricultural pest.

2. An alternative predator-prey model is often used, where the model includes an intraspecies competition term for the prey species. This model is given by the system of differential equations

$$\begin{aligned} \dot{H} &= a_1 H - a_2 HL - a_3 H^2 \\ \dot{L} &= -b1L + b_2 HL \end{aligned}$$

This system has an additional parameter, so how does this change the analysis of the model.

a. Find all equilibria for this model, then discuss the stability of these equilibria. Give the condition that makes one of the equilibria strictly positive in both the predator and prey populations and use this condition in your stability analysis. Characterize each of the equilibria (*e.g.*, stable node, saddle node, unstable spiral). Is this model structurally stable?

b. In this part of the problem you will repeat much of the work that was done in the lecture notes on the Lotka-Volterra model for the lynx and hare study. Use the information from the data to obtain estimates for the parameters, $H(0)$, $L(0)$, a_1, a_2, a_3, b_1, and b_2. For an initial estimate on a_3 use that the carrying capacity is at least twice the maximum observed population of hares. (In fact begin with the carrying capacity being exactly twice the maximum observed population of hares.) List all of your estimates and how you obtained them.

c. Create a MatLab program to find the least squares best fit to the data for lynx and hares from the Hudson Bay company. Write the sum of squares error between the model and the data. Give all of the parameters that you find and show graphs of both the data and the models. Produce a graph with the time evolution of both populations and another graph showing the phase portrait of the lynx and hare populations (including arrows to show the directionof the solution). Your graphs should be similar to the ones presented in the notes using this alternative model.

d. For the model that you found in Part c, find all equilibria and find the eigenvalues at those equilibria. Graph the nullclines, clearly labeling the equilibria. Show the direction field for this system of differential equations (being sure that the nullclines are clearly visible). Discuss the stability of each of the equilibria and predict what the model predicts will happen with the populations of the lynx and hares. Is this prediction reasonable? Clearly state your reasoning.