

**HINT:** 1. c. It appears that many students have the correct program, giving the correct result. However, students are having difficulty interpreting the results. The key to understanding these argument principle plots is blowing up the region near the origin and understanding what is contour means. You should create a cartoon expanding the size of the contour in this region and carefully trace what direction you are going around the origin. Note that a clockwise encirclement of the origin followed by a counterclockwise encirclement is equivalent to **no** encirclement. To improve your understanding, you may want to look at expanding rectangles for the cases when  $k = 1$  and  $k = 4$ . Rather than starting with the rectangle in the complex plane bounded by  $0 \leq x \leq 4$  and  $-3 \leq y \leq 3$ , you might find consider intervals like  $0 \leq x \leq 1$  and  $-1 \leq y \leq 1$  for the delay  $k = 1$  and possibly smaller like  $0 \leq x \leq 0.2$  and  $-0.5 \leq y \leq 0.5$  for the delay  $k = 4$ . Since the Hopf bifurcation lies fairly close to  $k = 4$ , you might get a better feeling of the encirclements by starting with  $k = 8$  and watch the changes as  $k$  is reduced to  $k = 4$ .

Hopefully, this will provide sufficient additional material to succeed with this part of the problem.