Math 342 A   Fall 2004

HW 9 $\boxed{9.6}$ 2, 3, 5, 6, 10, 11, 12, 13, 16, 17   8

(3) Let $f_1(x) = 1+x$ and $f_2(x) = x$. $\langle f_1, f_2 \rangle = \int_0^1 (1+x)x\,dx = \frac{x^2}{2} + \frac{x^3}{3}\Big|_0^1 = 5/6$

$\langle f_1, f_1 \rangle = \int_0^1 (1+x)^2 dx = \frac{(1+x)^3}{3}\Big|_0^1 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ $\langle f_2, f_2 \rangle = \int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$ $\|f_1\| = \sqrt{\frac{7}{3}}$, $\|f_2\| = \sqrt{1/3}$

$|\langle f_1, f_2 \rangle| = 5/6 \le \|f_1\| \cdot \|f_2\| = \frac{\sqrt{7}}{3}$ $(0.8333 \le 0.8819)$

(16) $\vec{v}_1 = [1, -1, 0]$, $\vec{v}_2 = [1, 1, 0]$, $\vec{v}_3 = [0, 1, 1]$ $\vec{u}_1 = \frac{\vec{v}_1}{\|v_1\|} = \frac{1}{\sqrt{2}}[1, -1, 0]$ Normalized

$\vec{u}_2 = \vec{v}_2 - \frac{\langle v_1, v_2 \rangle}{\|v_1\|^2} v_1 = \vec{v}_2$, $\vec{u}_2 = \frac{\vec{v}_2}{\|v_2\|} = \frac{1}{\sqrt{2}}[1, 1, 0]$, $\vec{u}_3 = \vec{v}_3 - \frac{\langle \vec{v}_3, \vec{v}_1 \rangle}{\|v_1\|^2} v_1 - \frac{\langle v_3, v_2 \rangle}{\|v_2\|^2} v_2$

$= [0, 1, 1] - (-\frac{1}{2})[1, -1, 0] - (\frac{1}{2})[1, 1, 0] = [0, 0, 1]$ $\therefore \vec{u}_3 = [0, 0, 1]$

$\boxed{10.1}$ 2, 3, 12   2

(2) Can solve $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$ or apply π rotation $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \to \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{bmatrix} a+1 & b \\ c & d+1 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow a = -1,\ b = 0,\ c = 0,\ d = -1$

$\boxed{10.2}$ 1, 2, 3, 4, 5, 9, 10, 17, 18, 19   8

(4) $\begin{vmatrix} 3-\lambda & -2 & -1 \\ 3 & -4-\lambda & -3 \\ 2 & -4 & -\lambda \end{vmatrix} = (3-\lambda)\big((-4-\lambda)(-\lambda) - 12\big) + 2(-3\lambda + 6) - \big(-12 + 2(4+\lambda)\big)$

$= (3-\lambda)(\lambda^2 + 4\lambda - 12) - 6\lambda + 12 + 4 - 2\lambda = -\lambda^3 - \lambda^2 + 16\lambda - 20$

$= -(\lambda^3 + \lambda^2 - 16\lambda + 20) = 0$ Note $\lambda = 2$ satisfies $\therefore -(\lambda - 2)(\lambda^2 + 3\lambda - 10) = -(\lambda-2)^2(\lambda+5) = 0$

$\therefore \lambda = 2, 2, -5$. For $\lambda_1 = -5$, $\begin{pmatrix} 8 & -2 & -1 \\ 3 & 1 & -3 \\ 2 & -4 & 5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \to \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \xi_1 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

For $\lambda_2 = 2$, $\begin{pmatrix} 1 & -2 & -1 \\ 3 & -6 & -3 \\ 2 & -4 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \left(\begin{array}{ccc|c} 1 & -2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$ $x_3 = t$, $x_2 = s$ $x_1 = 2s + t$ $\xi_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Thus, we have 3 lin. indep. e.f.'s.

(17) $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ $\lambda_1 = 5$, $\xi_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\lambda_2 = 1$, $\xi_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ with Maple

$18 + 25 = 43$