Math 342A Fall 2004

HW 8 $\boxed{9.4}$ 1, 3, 5, $\underline{7}$, 10, 11, $\underline{13}$, 14

7.
$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & -4 & -2 & 2 & 6 \\ 4 & 1 & -3 & 3 & -1 \end{array}\right] \xrightarrow[\substack{R_3-\frac{1}{2}R_1 \\ R_4-2R_1}]{R_2-\frac{1}{2}R_1} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & -3/2 & -1/2 & 1/2 & 2 \\ 0 & -9/2 & -3/2 & 3/2 & 7 \\ 0 & -1 & -1 & 1 & 3 \end{array}\right] \xrightarrow[-2R_3]{-2R_2} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 3 & 1 & -1 & -4 \\ 0 & 9 & 3 & -3 & -14 \\ 0 & -1 & -1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow[R_4+\frac{1}{3}R_2]{R_3-3R_2} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 3 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & -2/3 & 2/3 & 5/3 \end{array}\right] \xrightarrow[3R_4]{R_4 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 2 & 1 & -1 & 1 & -2 \\ 0 & 3 & 1 & -1 & -4 \\ 0 & 0 & -2 & 2 & 5 \\ 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

Rank$(A) = 3$, Rank$(A|b) = 4$

No solution.

13.
$$\det\begin{bmatrix} x & 1 & 0 & 1 \\ 1 & x & 1 & 0 \\ 0 & 1 & x & 1 \\ 1 & 0 & 1 & x \end{bmatrix} = x\begin{vmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{vmatrix} - \begin{vmatrix} 1 & 1 & 0 \\ 0 & x & 1 \\ 1 & 1 & x \end{vmatrix} - \begin{vmatrix} 1 & x & 1 \\ 0 & 1 & x \\ 1 & 0 & 1 \end{vmatrix}$$

$$= x\left(x\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 0 & x \end{vmatrix}\right) - \begin{pmatrix} x & 1 \\ 1 & x \end{pmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & x \end{vmatrix} - \begin{vmatrix} 1 & x \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$$

$$= x^2(x^2-1) - x^2 - (x^2-1) - 1 - 1 - (x^2-1) = x^4 - 4x^2 = 0 \quad \therefore \ x = 0, 0, \pm 2.$$

Nontrivial soln. when $x = 0, \pm 2$

$\boxed{9.5}$ $\underline{3}$, 4, 5, 6, 7, 9, $\underline{12}$, 13, 16

3. $u = a_3x^3 + a_2x^2 + a_1x + a_0$, $v = b_3x^3 + b_2x^2 + b_1x + b_0$, $w = c_3x^3 + c_2x^2 + c_1x + c_0$

$u+v = (a_3+b_3)x^3 + (a_2+b_2)x^2 + (a_1+b_1)x + (a_0+b_0) \in V$ closed under addition

$u+v = v+u$, so commutative, $(u+v)+w = u+(v+w)$, so associative

Take $a_3 = a_2 = a_1 = a_0$, then $u \equiv 0$ is the unique zero vector

If $b_3 = -a_3$, $b_2 = -a_2$, $b_1 = -a_1$, $b_0 = -a_0$, then $u+v = 0$ or $v = -u$.

$\alpha u = \alpha a_3x^3 + \alpha a_2x^2 + \alpha a_1x + \alpha a_0 \in V$, closed under scalar mult.

$(\alpha+\beta)u = \alpha u + \beta u$, $\alpha(u+v) = \alpha u + \alpha v$ and distributive

$\alpha(\beta u) = (\alpha\beta)u$ (associative)

$1 \cdot u = u$ (scalar identity) V has 4-dimensions

12.
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right] \xrightarrow[\substack{-R_2 \\ R_1+R_2}]{R_3+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_1-R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

Thus $[1,2,3] = -2[1,1,0] - [1,0,1] + 4[1,1,1]$

$\underline{13+20=33}$