Math 342A

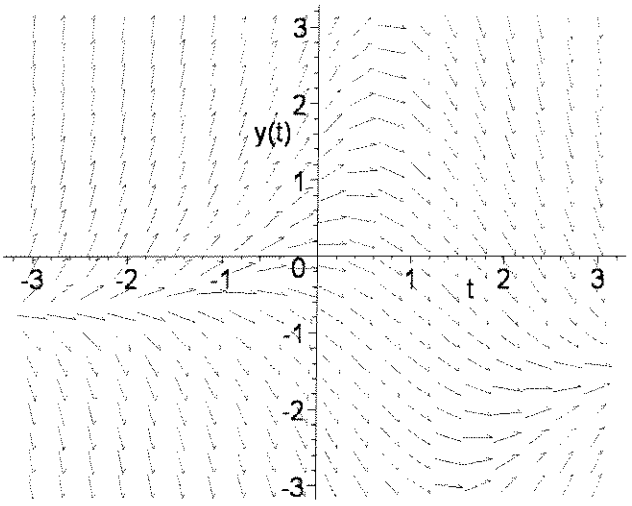
Fall 2004

HW 12 B&C p. 74 1, 2, 3, 4, 5, 6, Maple 7, 10, 13, 14

3. a. $y' = -(y-1)(y-2)$ Equil at $y=1, y=2$. slope negative $y<1$ or $y>2$, slope positive $1<y<2$

7. `> de:=diff(y(t),t)=(1-t)*y(t)-t;DEplot(de,y(t),t=-3..3,y=-3..3);`

$$de := \frac{\partial}{\partial t} y(t) = (1-t)\,y(t) - t$$



B&C p129 1 $y' = -y$, $y(0)=1$, $h=0.1$ $y(x) = e^{-x}$

a. Euler

t	Actual	Euler	RK4
0	1	1	1
0.1	0.904837	0.9	0.904838
0.2	0.818731	0.81	0.818731
0.3	0.740818	0.729	0.740818
0.4	0.67032	0.6561	0.67032
0.5	0.606531	0.59049	0.606531
0.6	0.548812	0.531441	0.548812
0.7	0.496585	0.478297	0.496586
0.8	0.449329	0.430467	0.449329
0.9	0.40657	0.38742	0.40657
1	0.367879	0.348678	0.36788

11.3 2, 3, 5, 6, 7, 9, 11, 12, 17 7+10

(5) a. $y''-4y=0$, $\lambda^2-4=0$ $y(x) = c_1 e^{2x} + c_2 e^{-2x}$

b. $y''+2y'+4=0$, $\lambda^2+2\lambda+4=0$ $\lambda = -1 \pm i\sqrt{3}$, $y(x) = e^{-x}\left(c_1\cos(\sqrt{3}x) + c_2\sin(\sqrt{3}x)\right)$

c. $y''+9y=0$, $\lambda^2+9=0$ $\lambda = \pm 3i$, $y(x) = c_1\cos(3x) + c_2\sin(3x)$

(12) a. $x^2y'' + xy' - 4y = 0$, $y = x^2u$, $y' = x^2u' + 2xu$, $y'' = x^2u'' + 4xu' + 2u$

$\Rightarrow x^2(x^2u'' + 4xu' + 2u) + x(x^2u' + 2xu) - 4x^2u = x^4u'' + 5x^3u' = 0 \quad \therefore u'' + \frac{5}{x}u' = 0$, $w = u'$

$w' + \frac{5}{x}w = 0$, $\mu(x) = \exp\int\frac{5}{x}dx = x^5 \quad \therefore \frac{d}{dx}(wx^5) = 0 \Rightarrow w(x) = c_1x^{-5} = u' \Rightarrow u(x) = \hat{c}_1x^{-4} + c_2$

$\therefore y(x) = x^2(\hat{c}_1x^{-4} + c_2) = \hat{c}_1x^{-2} + c_2x^2$

15 + 25 = 40

8 + 15

7 + 10