Math 342A

Fall 2004

HW6 $\boxed{9.1}$ 1, 3, 6, 7, 21

7. $\begin{vmatrix} x & 1 & 1 & 1 \\ 1 & x & 0 & 0 \\ 1 & 0 & x & 0 \\ 1 & 0 & 0 & x \end{vmatrix} = -\begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} + x \begin{vmatrix} x & 1 & 1 \\ 1 & x & 0 \\ 1 & 0 & x \end{vmatrix} = -\begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} + x\left(-\begin{vmatrix} 1 & 1 \\ 0 & x \end{vmatrix} + x\begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}\right)$

$= -x^2 + x(-x + x + x(x^2-1)) = x^4 - 3x^2$

$x = 0, 0, \pm\sqrt{3}$.

$4+5=9$

HW7 $\boxed{9.2}$ 2, 4, 7, 10

7. $\left|\begin{matrix} 2 & -4 & 1 & -3 & : & 6 \\ 1 & -2 & 3 & 6 & : & 2 \end{matrix}\right| \xrightarrow[R_2 - R_1]{\frac{1}{2}R_1} \left|\begin{matrix} 1 & -2 & \frac{1}{2} & -\frac{3}{2} & : & 3 \\ 0 & 0 & \frac{5}{2} & \frac{15}{2} & : & -1 \end{matrix}\right| \xrightarrow{\frac{2}{5}R_2} \left|\begin{matrix} 1 & -2 & \frac{1}{2} & -\frac{3}{2} & : & 3 \\ 0 & 0 & 1 & 3 & : & -\frac{2}{5} \end{matrix}\right|$

$x_4 = s, \quad x_3 = -3s - \frac{2}{5}, \quad x_2 = t, \quad x_1 = 2t + \frac{1}{2}(3s + \frac{2}{5}) + \frac{3}{2}s + 3$

$\therefore x_1 = 2t + 3s + 16/5$

$\boxed{9.3}$ 2, 4, 5, 7, 8, 9, 12, 16, 19, 24

8. $C_3 C_3^T = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad \sigma_v \sigma_v^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

All orthogonal

$\sigma_v'(\sigma_v')^T = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\quad \sigma_v''(\sigma_v'')^T = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

16. $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad A^{-1} = \frac{1}{-2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} \quad (A^{-1})^{-1} = -2\begin{pmatrix} -\frac{1}{2} & -1 \\ -\frac{3}{2} & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$

$(A^T)^{-1} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}^{-1} = \frac{1}{-2}\begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -2 & \frac{3}{2} \\ 1 & -\frac{1}{2} \end{pmatrix} = (A^{-1})^T$.

$\det(A^{-1}) = -\frac{1}{2} \quad (\det(A))^{-1} = (-2)^{-1} = \det(A^{-1})$

$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 10 & 7 \\ 22 & 15 \end{pmatrix} \quad \det(AB) = -4 \quad \therefore (AB)^{-1} = -\frac{1}{4}\begin{pmatrix} 15 & -7 \\ -22 & 10 \end{pmatrix} \quad B^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

$B^{-1}A^{-1} = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{15}{4} & \frac{7}{4} \\ \frac{11}{2} & -\frac{5}{2} \end{pmatrix} = (AB)^{-1}$

$11 + 15 = 26$