Math 342A

Fall 2004

HW1 $\boxed{5.1}$ 1, 3, 7, 12

(12) Perpendicular distance from (4, 3) to the line $x + 2y - 4 = 0$. slope $\perp$, $m_\perp = 2$

$\therefore\ y - 3 = 2(x-4) \Rightarrow y = 2x - 5$. Intersection $5x - 14 = 0$, $x = 14/5$, $y = 3/5$

$d = ((4 - 14/5)^2 + (3 - 3/5)^2)^{1/2} = \frac{6}{5}\sqrt{5}$

$\boxed{5.2}$ 1, 4, 7, 11, 13, 15, 17, 20

(4) $\vec{a}(t) = 3t\vec{i} - 2t^2\vec{j}$, $\vec{r}(0) = 2\vec{i}$, $\vec{v}(0) = \vec{i} + \vec{j}$, $\vec{v}(t) = \left(\frac{3}{2}t^2 + c_1\right)\vec{i} - \left(\frac{2}{3}t^3 + c_2\right)\vec{j}$

I.C. $\Rightarrow \vec{v}(t) = \left(\frac{3}{2}t^2 + 1\right)\vec{i} - \left(\frac{2}{3}t^3 - 1\right)\vec{j}$, $\vec{r}(t) = \left(\frac{1}{2}t^3 + t + c_4\right)\vec{i} - \left(\frac{1}{6}t^4 - t + c_4\right)\vec{j}$

I.C. $\Rightarrow \vec{r}(t) = \left(\frac{1}{2}t^3 + t + 2\right)\vec{i} - \left(\frac{1}{6}t^4 - t\right)\vec{j}$

(20) $\vec{r}(t) = t\cos(t)\,\vec{i} + t\sin(t)\,\vec{j}$, $\vec{v}(t) = (\cos(t) - t\sin(t))\vec{i} + (\sin(t) + t\cos(t))\vec{j}$, $\vec{v}(0) = \vec{i}$

$\vec{a}(t) = (-2\sin(t) - t\cos(t))\vec{i} + (2\cos(t) - t\sin(t))\vec{j}$, $\vec{a}(0) = 2\vec{j}$

$\frac{ds}{dt} = |\vec{v}(t)| = \left((\cos(t) - t\sin(t))^2 + (\sin(t) + t\cos(t))^2\right)^{1/2} = (1+t^2)^{1/2}$ $\quad \frac{d^2s}{dt^2} = a_T = t(1+t^2)^{-1/2}$

$a_T(0) = 0$, Since $\vec{N}(0) = \vec{j}$, $a_N(0) = 2$.

$\boxed{5.3}$ 3, 6, 10, 12, 18

(6) $\vec{u} = [1,1,1]$, $\vec{v} = [2,3,5]$, $A = |\vec{u} \times \vec{v}| = \left\| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 3 & 5 \end{matrix} \right\| = |2\vec{i} - 3\vec{j} + \vec{k}| = \sqrt{14}$

13 + 20/33

HW2 $\boxed{5.4}$ 2, 3, 6, 9, 15, 16, 18, 20, 23, 24

(9) $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$, $\vec{v}(t) = \vec{i} + 2t\vec{j} + 3t^2\vec{k}$, $v = (1 + 4t^2 + 9t^4)^{1/2}$

$\vec{T}(t) = \dfrac{\vec{i} + 2t\vec{j} + 3t^2\vec{k}}{(1 + 4t^2 + 9t^4)^{1/2}}$

(23) $\vec{r}(t) = t\vec{i} + t^2\vec{j} + \frac{2}{3}t^3\vec{k}$ $\quad \vec{v}(t) = \vec{i} + 2t\vec{j} + 2t^2\vec{k}$, $v = (1 + 4t^2 + 4t^4)^{1/2} = 1 + 2t^2$

$\vec{T}(t) = \dfrac{\vec{i} + 2t\vec{j} + 2t^2\vec{k}}{1 + 2t^2}$ $\quad \dfrac{d\vec{T}}{dt} = \dfrac{-4t\vec{i} + (2 - 4t^2)\vec{j} + 4t\vec{k}}{(1+2t^2)^2}$ $\quad \left|\dfrac{d\vec{T}}{dt}\right| = \dfrac{(16t^2 + 4 - 16t^2 + 16t^4 + 16t^2)^{1/2}}{(1+2t^2)^2}$

$\Rightarrow \left|\dfrac{d\vec{T}}{dt}\right| = \dfrac{2}{1+2t^2}$ $\quad \therefore\ \vec{N} = \dfrac{d\vec{T}/dt}{|d\vec{T}/dt|} = \dfrac{-2t\vec{i} + (1 - 2t^2)\vec{j} + 2t\vec{k}}{1 + 2t^2}$, $\kappa = \frac{1}{v}\left|\dfrac{d\vec{T}}{dt}\right| = \dfrac{2}{(1+2t^2)^2}$

$\vec{B} = \vec{T} \times \vec{N} = \left| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2t & 2t^2 \\ -2t & 1-2t^2 & 2t \end{matrix} \right| \dfrac{1}{(1+2t^2)^2} = \dfrac{1}{(1+2t^2)^2}\left((2t^2 + 4t^4)\vec{i} - (2t + 4t^3)\vec{j} + (1 + 2t^2)\vec{k}\right)$

$\vec{B} = \dfrac{2t^2\vec{i} - 2t\vec{j} + \vec{k}}{1 + 2t^2}$ $\quad \dfrac{d\vec{B}}{dt} = \dfrac{4t\vec{i} + (4t^2 - 2)\vec{j} - 4t\vec{k}}{(1+2t^2)^2}$, $\quad \dfrac{d\vec{B}}{ds} = \dfrac{1}{v}\dfrac{d\vec{B}}{dt} = -\tau N$

$\therefore\ \tau = \dfrac{2}{(1+2t^2)^2}$

$\boxed{5.5}$ 1, 3, 5, 8, 9, 10

(8) $\vec{n}_1 = [1, -1, 2]$, $\vec{n}_2 = [3, 1, 2]$ $\quad \vec{n}_1 \times \vec{n}_2 = \left| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ 3 & 1 & 2 \end{matrix} \right| = -4\vec{i} + 4\vec{j} + 4\vec{k}$ $\quad$ line direction $\vec{l} = [1, -1, -1]$

pt. Let $x = 0$, then $-y + 2z = 3$ & $y + 2z = 7 \Rightarrow y = 2$, $z = 5/2$ $\therefore$ line satisfies $\dfrac{x}{1} = \dfrac{y-2}{-1} = \dfrac{z - 5/2}{-1}$

13 + 15/28