

HWS 7.4 2, 4, 7, 8, 9, 11

8.  $\vec{F} = xy^2\vec{i} + x^2y\vec{j} + y\vec{k}$  and  $S_1 = \{(x,y,z) : x^2 + y^2 = 1, -1 \leq z \leq 1\}$ ,  $S_2 = \{(x,y,z) : x^2 + y^2 < 1, z = -1\}$

$S_3 = \{(x,y,z) : x^2 + y^2 < 1, z = 1\}$ . For  $S_1$ ,  $\vec{r}_1(u,v) = \cos u\vec{i} + \sin u\vec{j} + v\vec{k}$ ,  $0 \leq u \leq 2\pi$ ,  $-1 \leq v \leq 1$   
 For  $S_2$ ,  $\vec{r}_2(u,v) = u\cos v\vec{i} + u\sin v\vec{j} - \vec{k}$ ,  $0 \leq u \leq 1$ ,  $0 \leq v \leq 2\pi$ . For  $S_3$ ,  $\vec{r}_3(u,v) = u\cos v\vec{i}$

$+ u\sin v\vec{j} + \vec{k}$ ,  $0 \leq u \leq 1$ ,  $0 \leq v \leq 2\pi$ .  $\vec{N}_1 = \vec{r}_{1u} \times \vec{r}_{1v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos u\vec{i} + \sin u\vec{j}$   
 $\vec{N}_2 = \vec{r}_{2u} \times \vec{r}_{2v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = u\vec{k}$  oriented  $\vec{N}_2 = -u\vec{k}$ , similarly  $\vec{N}_3 = \vec{F}_{3u} \times \vec{r}_{3v} = u\vec{k}$

Let  $S = S_1 + S_2 + S_3$   $\iint_S \vec{F} \cdot \vec{n} \, dS = \int_{-1}^1 \int_0^{2\pi} (\cos u \sin^2 u \cdot \cos u + \cos^2 u \sin u \cdot \sin u) \, du \, dv$   
 $+ \int_0^{2\pi} \int_0^1 u \sin(v) (-u) \, dv \, du + \int_0^{2\pi} \int_0^1 u \sin(v) (u) \, dv \, du = 4 \int_0^{2\pi} \sin^2 u \cos^2 u \, du = \int_0^{2\pi} (1 - \cos^2(2u)) \, du$   
 $= 2\pi - \int_0^{2\pi} \frac{1 + \cos(4u)}{2} \, du = \pi$ ,  $\vec{\nabla} \cdot \vec{F} = y^2 + x^2$   $\iiint_V \vec{\nabla} \cdot \vec{F} \, dV = \int_{-1}^1 \int_0^{2\pi} \int_0^1 r^2 \cdot r \, dr \, d\theta \, dz$   
 $= 2 \cdot 2\pi \cdot \frac{r^4}{4} \Big|_0^1 = \pi$

7.5 2, 4, 7, 10, 12, 14, 21

2.  $\oint_C \vec{F} \cdot d\vec{r}$  with  $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ . Let  $\vec{r} = \cos(t)\vec{i} + \sin(t)\vec{j}$ ,  $0 \leq t \leq 2\pi$ .  
 $\int_0^{2\pi} \sin(t) (-\sin(t)) \, dt = -\int_0^{2\pi} \frac{1 - \cos(2t)}{2} \, dt = -\pi$ .  $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{vmatrix} = -\vec{i} - \vec{j} - \vec{k}$

$\vec{r}(u,v) = u\cos(v)\vec{i} + u\sin(v)\vec{j} + (1-u^2)\vec{k}$   $0 \leq u \leq 1$ ,  $0 \leq v \leq 2\pi$   $\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos(v) & \sin(v) & -2u \\ -u\sin(v) & u\cos(v) & 0 \end{vmatrix} = -2u^2\cos(v)\vec{i} + 2u^2\sin(v)\vec{j} + u\vec{k}$   
 $\iint_S \vec{\nabla} \times \vec{F} \cdot \vec{n} \, dS = \int_0^{2\pi} \int_0^1 (2u^2\cos(v) - 2u^2\sin(v) - u) \, dv \, du = -2\pi \left(\frac{u^3}{3}\right) \Big|_0^1 = -\pi$

14.  $\vec{F} = (x+y)\vec{i} + (z-2x+y)\vec{j} + (y-z)\vec{k}$ ,  $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & z-2x+y & y-z \end{vmatrix} = 0\vec{i} + 0\vec{j} - 3\vec{k}$   
 $\vec{n} = \vec{k}$   
 $\oint_C \vec{F} \cdot d\vec{r} = \iint_S \vec{\nabla} \times \vec{F} \cdot \vec{n} \, dS = \iint_S (-3) \, dS = -3\pi$  as  $S$  is area of unit circle

10 + 15 = 25