Math 342A — Fall 2004

HW5 $\boxed{7.4}$ 2, 4, 7, 8, 9, 11

8. $\vec{F} = xy^2\vec{i} + x^2y\vec{j} + y\vec{k}$ and $S_1 = \{(x,y,z) : x^2+y^2=1, -1<z<1\}$, $S_2 = \{(x,y,z) : x^2+y^2<1, z=-1\}$ $S_3 = \{(x,y,z) : x^2+y^2<1, z=1\}$. For S_1, $\vec{r}_1(u,v) = \cos u\,\vec{i} + \sin u\,\vec{j} + v\vec{k}$, $0 \le u \le 2\pi$, $-1 \le v \le 1$

For S_2, $\vec{r}_2(u,v) = u\cos v\,\vec{i} + u\sin v\,\vec{j} - \vec{k}$, $0 \le u \le 1$, $0 \le v \le 2\pi$. For S_3, $\vec{r}_3(u,v) = u\cos v\,\vec{i} + u\sin v\,\vec{j} + \vec{k}$, $0 \le u \le 1$, $0 \le v \le 2\pi$. $\vec{N}_1 = \vec{r}_{1u} \times \vec{r}_{1v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos u\,\vec{i} + \sin u\,\vec{j}$

$\vec{N}_2 = \vec{r}_{2u} \times \vec{r}_{2v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u\sin v & u\cos v & 0 \end{vmatrix} = u\vec{k}$ oriented $\vec{N}_2 = -u\vec{k}$, similarly $\vec{N}_3 = \vec{r}_{3u} \times \vec{r}_{3v} = u\vec{k}$

Let $S = S_1 + S_2 + S_3$ $\iint_S \vec{F}\cdot\vec{n}\,dS = \int_{-1}^{1}\int_0^{2\pi} (\cos u \sin^2 u \cdot \cos u + \cos^2 u \sin u \cdot \sin u)\,du\,dv$

$+ \int_0^1\int_0^{2\pi} u\sin(v)(-u)\,dv\,du + \int_0^1\int_0^{2\pi} u\sin(v)(u)\,dv\,du = 4\int_0^{2\pi} \sin^2 u\cos^2 u\,du = \int_0^{2\pi}(1-\cos^2(2u))\,du$

$= 2\pi - \int_0^{2\pi} \frac{1+\cos(4u)}{2}\,du = \pi$. $\vec{\nabla}\cdot\vec{F} = y^2 + x^2$ $\iiint_V \vec{\nabla}\cdot\vec{F}\,dV = \int_{-1}^{1}\int_0^{2\pi}\int_0^1 r^2\cdot r\,dr\,d\theta\,dz$

$= 2\cdot 2\pi \cdot \frac{r^4}{4}\Big|_0^1 = \pi$

$\boxed{7.5}$ 2, 4, 7, 10, 12, 14, 21

2. $\oint_C \vec{F}\cdot d\vec{r}$ with $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$. Let $\vec{r} = \cos(t)\vec{i} + \sin(t)\vec{j}$, $0 \le t \le 2\pi$.

$\int_0^{2\pi} \sin(t)(-\sin(t))\,dt = -\int_0^{2\pi} \frac{1-\cos(2t)}{2}\,dt = -\pi$. $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & z & x \end{vmatrix} = -\vec{i} - \vec{j} - \vec{k}$

$\vec{r}(u,v) = u\cos(v)\vec{i} + u\sin(v)\vec{j} + (1-u^2)\vec{k}$ $\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos(v) & \sin(v) & -2u \\ -u\sin(v) & u\cos(v) & 0 \end{vmatrix} = -2u^2\cos(v)\vec{i} + 2u^2\sin(v)\vec{j} + u\vec{k}$

$0 \le u \le 1$, $0 \le v \le 2\pi$

$\iint_S \vec{\nabla}\times\vec{F}\cdot\vec{n}\,dS = \int_0^1\int_0^{2\pi} (\overset{0}{2u^2\cos(v)} - \overset{0}{2u^2\sin(v)} - u)\,dv\,du = -2\pi\left(\frac{u^2}{2}\right)\Big|_0^1 = -\pi$

14. $\vec{F} = (x+y)\vec{i} + (z-2x+y)\vec{j} + (y-z)\vec{k}$, $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y & z-2x+y & y-z \end{vmatrix} = 0\vec{i} + 0\vec{j} - 3\vec{k}$

$\vec{n} = \vec{k}$

$\oint_C \vec{F}\cdot d\vec{r} = \iint_S \vec{\nabla}\times\vec{F}\cdot\vec{n}\,dS = \iint_S (-3)\,dS = -3\pi$ as S is area of unit circle

$10 + 15 = 25$