

HW3 6.5 4, 5, 9

(9) $V = \frac{4}{3}\pi abc$, $dV = \frac{4}{3}\pi(kc da + ec db + ab dc)$ Max deviation (Min is negative)

$$dV = \frac{4}{3}\pi(80(.05) + 80(.05) + 100(.05)) = \frac{52\pi}{3} = 54.45 \text{ (uncertainty in volume)}$$

$$\text{Rel. uncertainty} = \frac{dV}{V} = \frac{52\pi/3}{4\pi/3(800)} = \frac{13}{800} = 0.01625 \approx 1.625\%$$

6.6 2, 5, 10, 11, 12, 14

(12) $z = 1000 e^{-(2x^2+y^2)/200}$, $\vec{\nabla} z = (-20x\vec{i} - 10y\vec{j})e^{-(2x^2+y^2)/200}$

$$\vec{\nabla} z(5, 10) = (-100\vec{i} - 100\vec{j})e^{-3/4} = \text{direction of max ascent} \therefore \text{steepest descent} = -\vec{\nabla} z$$

$$= 100e^{-3/4}(\vec{i} + \vec{j}). \text{Rate of descent} = |\vec{\nabla} z| = 100e^{-3/4}\sqrt{2} = 66.803$$

7.1 2, 3, 5, 6, 7, 8, 9, 12, 13

(3) $\vec{A} = xy^2\vec{i} + 2xyz\vec{j} - x^2z\vec{k}$, $\text{div}(\vec{A}) = \vec{\nabla} \cdot \vec{A} = y^2 + 2xz - x^2$

$$\text{curl}(\vec{A}) = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ xy^2 & 2xyz & -x^2z \end{vmatrix} = -2xy\vec{i} + 2xz\vec{j} + (2yz - 2xy)\vec{k}$$

(8) $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $r = (x^2 + y^2 + z^2)^{1/2}$, $\vec{\nabla}\left(\frac{1}{r}\right) = (x^2 + y^2 + z^2)^{-3/2}(-x\vec{i} - y\vec{j} - z\vec{k}) = -\frac{\vec{r}}{r^3}$

$$\vec{\nabla} \cdot \left(\vec{\nabla}\left(\frac{1}{r}\right)\right) = \frac{-(x^2 + y^2 + z^2)^{3/2} + 3x^2(x^2 + y^2 + z^2)^{1/2}}{(x^2 + y^2 + z^2)^3} + \frac{-(x^2 + y^2 + z^2)^{3/2} + 3y^2(x^2 + y^2 + z^2)^{1/2}}{(x^2 + y^2 + z^2)^3} + \frac{-(x^2 + y^2 + z^2)^{3/2} + 3z^2(x^2 + y^2 + z^2)^{1/2}}{(x^2 + y^2 + z^2)^3}$$

$$= (x^2 + y^2 + z^2)^{-5/2}[-3(x^2 + y^2 + z^2) + 3x^2 + 3y^2 + 3z^2] = 0 \therefore \nabla^2\left(\frac{1}{r}\right) = 0 \quad (r \neq 0)$$

$$14 + 20 = 34$$

HW4 7.2 1, 3, 5, 6, 8, 11, 12, 14, 16, 18

(5) $\vec{A} = x^2y\vec{i} + y^2z\vec{j} + z^2x\vec{k}$, $C: \vec{r}(t) = t\vec{i} + t\vec{j} + 3t\vec{k}$, $t \in [0, 1]$

$$\int_C \vec{A} \cdot d\vec{r} = \int_0^1 [t^3 + 3t^3 + 9t^3 \cdot 3] dt = \int_0^1 31t^3 dt = \frac{31}{4}t^4 \Big|_0^1 = \frac{31}{4}$$

(16) $P = y - x^2$, $Q = 2x + y^2$, $R = z(x, y)$, $1 \leq x \leq 4$, $1 \leq y \leq 3$. $\int_C P dx + Q dy = \int_1^4 (1 - x^2) dx + \int_1^3 (8 + y^4) dy + \int_4^1 (3 - x^2) dx$

$$+ \int_3^1 (2 + y^4) dy = (x - \frac{x^3}{3}) \Big|_1^4 + (8y + \frac{y^5}{5}) \Big|_1^3 + (3x - \frac{x^3}{3}) \Big|_4^1 + (2y + \frac{y^5}{5}) \Big|_3^1 = 3 + 16 - 9 - 4 = 6$$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_1^4 \int_1^3 (2 - 1) dx dy = 6$$

7.3 1, 4, 5, 7, 9, 10, 12

(10) Define the surface S , $\vec{r}(u, v) = a \cos(u) \sin(v)\vec{i} + a \sin(u) \sin(v)\vec{j} + a \cos(v)\vec{k}$, $0 \leq u \leq \frac{\pi}{2}$, $0 \leq v \leq \frac{\pi}{2}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a \sin(u) \sin(v) & a \cos(u) \sin(v) & 0 \\ a \cos(u) \cos(v) & a \sin(u) \cos(v) & -a \sin(v) \end{vmatrix} = -a^2 \cos(u) \sin^2(v)\vec{i} - a^2 \sin(u) \sin^2(v)\vec{j} - a^2 \sin(v) \cos(v)\vec{k}$$

$$|\vec{r}_u \times \vec{r}_v| = a^2 (\cos^2(u) \sin^4(v) + \sin^2(u) \sin^4(v) + \sin^2(v) \cos^2(v))^{1/2} = a^2 \sin(v)$$

$$\iint_S \vec{x} \cdot d\vec{S} = \int_0^{\pi/2} \int_0^{\pi/2} a \cos(u) \sin(v) (a^2 \sin(v)) du dv = a^3 \int_0^{\pi/2} \sin^2(v) \sin(u) \Big|_0^{\pi/2} dv = a^3 \int_0^{\pi/2} \frac{1 - \cos(2v)}{2} dv = \frac{a^3 \pi}{4}$$

$$14 + 15 = 29$$