Math 342A　　　　　　　　　　　　Fall 2004

HW3 $\boxed{6.5}$ 4, 5, 9

(9) $V = \frac{4}{3}\pi abc$, $dV = \frac{4}{3}\pi\left(bc\, da + ac\, db + ab\, dc\right)$ Max deviation (Min is negative)

$dV = \frac{4}{3}\pi\left(80(.05) + 80(.05) + 100(.05)\right) = \frac{52\pi}{3} = 54.45$ (uncertainty in volume)

Rel. uncertainty $= \frac{dV}{V} = \frac{52\pi/3}{4\pi/3(800)} = \frac{13}{800} = 0.01625$ or 1.625%.

$\boxed{6.6}$ 2, 5, 10, 11, 12, 14

(12) $z = 1000\, e^{-(2x^2+y^2)/200}$, $\vec{\nabla} z = \left(-20x\vec{i} - 10y\vec{j}\right)e^{-(2x^2+y^2)/200}$

$\vec{\nabla} z(5,10) = \left(-100\vec{i} - 100\vec{j}\right)e^{-3/4}$ = direction of max ascent $\therefore$ steepest descent $= -\vec{\nabla} z$

$= 100e^{-3/4}\left(\vec{i} + \vec{j}\right)$. Rate of descent $= |\vec{\nabla} z| = 100e^{-3/4}\sqrt{2} = 66.803$

$\boxed{7.1}$ 2, 3, 5, 6, 7, 8, 9, 12, 13

(3) $\vec{A} = xy^2\vec{i} + 2xyz\vec{j} - x^2z\vec{k}$, $\operatorname{div}(\vec{A}) = \vec{\nabla}\cdot\vec{A} = y^2 + 2xz - x^2$,

$$\operatorname{curl}(\vec{A}) = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2 & 2xyz & -x^2z \end{vmatrix} = -2xy\vec{i} + 2xz\vec{j} + (2yz - 2xy)\vec{k}$$

(8) $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, $r = (x^2+y^2+z^2)^{1/2}$, $\vec{\nabla}\left(\frac{1}{r}\right) = (x^2+y^2+z^2)^{-3/2}\left(-x\vec{i} - y\vec{j} - z\vec{k}\right) = -\vec{r}/r^3$

$$\vec{\nabla}\cdot\left(\vec{\nabla}(1/r)\right) = \frac{-(x^2+y^2+z^2)^{3/2} + x\,3x(x^2+y^2+z^2)^{1/2}}{(x^2+y^2+z^2)^3} + \frac{-(x^2+y^2+z^2)^{3/2} + 3y^2(x^2+y^2+z^2)^{1/2}}{(x^2+y^2+z^2)^3} + \frac{-(x^2+y^2+z^2)^{3/2} + 3z^2(x^2+y^2+z^2)^{1/2}}{(x^2+y^2+z^2)^3}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[-3(x^2+y^2+z^2) + 3x^2+3y^2+3z^2\right] = 0 \quad \therefore \nabla^2(1/r) = 0 \quad (r \neq 0).$$

$14 + 20 = 34$

HW4 $\boxed{7.2}$ 1, 3, 5, 6, 8, 11, 12, 14, 16, 18

(5) $\vec{A} = x^2y\vec{i} + y^2z\vec{j} + z^2x\vec{k}$, $C: \vec{r}(t) = t\vec{i} + t\vec{j} + 3t\vec{k}$, $t \in [0,1]$

$$\int_C \vec{A}\cdot d\vec{r} = \int_0^1 \left[t^3 + 3t^3 + 9t^3\cdot 3\right]dt = \int_0^1 31t^3\,dt = \frac{31}{4}t^4\Big|_0^1 = 31/4$$

(16) $P = y - x^2$, $Q = 2x + y^2$, $R = \{(x,y) \mid 1 \le x \le 4,\ 1 \le y \le 3\}$ $\int_C P\,dx + Q\,dy = \int_1^4 (1-x^2)\,dx + \int_1^3 (8+y^2)\,dy + \int_4^1 (3-x^2)\,dx$

$$+ \int_3^1 (2+y^2)\,dy = \left(x - \frac{x^3}{3}\right)\Big|_1^4 + \left(8y + \frac{y^3}{3}\right)\Big|_1^3 + \left(3x - \frac{x^3}{3}\right)\Big|_4^1 + \left(2y + \frac{y^3}{3}\right)\Big|_3^1 = 3 + 16 - 9 - 4 = 6$$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA = \int_1^3\int_1^4 (2-1)\,dx\,dy = 6$$

$\boxed{7.3}$ 1, 4, 5, 7, 9, 10, 12

(10) Define the surface S, $\vec{r}(u,v) = a\cos(u)\sin(v)\vec{i} + a\sin(u)\sin(v)\vec{j} + a\cos(v)\vec{k}$, $0 \le u \le \pi/2$, $0 \le v \le \pi/2$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a\sin(u)\sin(v) & a\cos(u)\sin(v) & 0 \\ a\cos(u)\cos(v) & a\sin(u)\cos(v) & -a\sin(v) \end{vmatrix} = -a^2\cos(u)\sin^2(v)\vec{i} - a^2\sin(u)\sin^2(v)\vec{j} - a^2\sin(v)\cos(v)\vec{k}$$

$$|\vec{r}_u \times \vec{r}_v| = a^2\left(\cos^2(u)\sin^4(v) + \sin^2(u)\sin^4(v) + \sin^2(v)\cos^2(v)\right)^{1/2} = a^2\sin(v)$$

$$\iint_S x\,dS = \int_0^{\pi/2}\int_0^{\pi/2} a\cos(u)\sin(v)\left(a^2\sin(v)\right)du\,dv = a^3\int_0^{\pi/2}\sin^2(v)\ \sin(u)\Big|_0^{\pi/2}dv = a^3\int_0^{\pi/2}\frac{1-\cos(2v)}{2}\,dv = \frac{a^3\pi}{4}$$

$14 + 15 = 29$