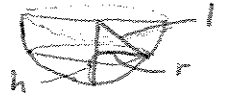


HW11 (11.1) 1, 2, 3, 4, 5, 6, 8, 9, 14, 15, 16

(4)  $\frac{dT}{dt} = -k(T - 20)$ ,  $z = T - 20 \Rightarrow \frac{dz}{dt} = -kz$ ,  $z(0) = T(0) - 20 = 180 \Rightarrow z(t) = T(t) - 20 = 180e^{-kt}$   
 $T(t) = 20 + 180e^{-kt}$ ,  $T(6) = 100 \Rightarrow 20 + 180e^{-6k} = 100 \Rightarrow e^{-6k} = \frac{80}{180} \Rightarrow k = \frac{1}{6} \ln(\frac{9}{4}) \approx 0.13516$   
 $T(t) = 35 = 20 + 180e^{-kt} \Rightarrow 180e^{-kt} = 15 \Rightarrow e^{-kt} = \frac{1}{12} \Rightarrow t = \frac{1}{k} \ln(12) \approx 26.5 \text{ min} \Rightarrow 20.5 \text{ min later}$

(15) From 14,  $\frac{dh}{dt} = -\frac{ac(2gh)^{1/2}}{A(h)}$ ,  $a = 10^{-4}$ ,  $c = 0.6$ ,  $g = 9.81$ .  $A(h) = \pi r^2$ , but



$$r^2 = 1 - (1-h)^2 = 2h - h^2, \quad \frac{dh}{dt} = -\frac{ac(2gh)^{1/2}}{\pi(2h-h^2)} = \frac{-6 \times 10^{-5} (19.62)^{1/2} h^{1/2}}{\pi(2h-h^2)}$$

$$\int (2h^{1/2} - h^{3/2}) dh = -8.958 \times 10^{-5} \int dt \Rightarrow \frac{4}{3} h^{3/2} - \frac{2}{5} h^{5/2} = -8.958 \times 10^{-5} t + C \Leftrightarrow 20 h^{3/2}(t) - 6 h^{5/2}(t) = -0.001269 t + C$$

$$h(0) = 1 \Rightarrow 20 - 6 = C \Rightarrow 20 h^{3/2}(t) - 6 h^{5/2}(t) = -0.001269 t + 14. \quad h(t) = 0 \Rightarrow 0.001269 t = 14 \Rightarrow$$

$$t = 11033 \text{ sec} \approx 3.06 \text{ hr.}$$

(11.2) 1, 2, 3, 4, 5, 6, 8, 10, 13, 15, 16, 20

(3) a.  $y' + \frac{1}{x}y = 2$ ,  $y(2) = 2$ ,  $\mu(x) = \exp(\int \frac{1}{x} dx) = x$ ,  $\frac{d}{dx}(xy) = 2x$ ,  $xy(x) = x^2 + C \Rightarrow y(x) = x + \frac{C}{x}$ ,  $y(2) = 2$   
 $\Rightarrow C = 0 \Rightarrow y(x) = x$

b.  $y' + \tan(x)y = \cos^2 x$ ,  $y(0) = -1$ ,  $\mu(x) = \exp(\int \frac{\sin(x)}{\cos(x)} dx) = \frac{1}{\cos(x)}$ ,  $\frac{d}{dx}(\frac{y}{\cos(x)}) = \cos(x)$ ,  $\frac{y(x)}{\cos(x)} = \sin(x) + C$

$$y(x) = \cos(x)\sin(x) + C \cos(x), \quad y(0) = -1 = C \Rightarrow y(x) = \cos(x)\sin(x) - \cos(x)$$

(15) a.  $\frac{du}{dt} + 2tu = 4t$ ,  $\mu(t) = \exp(\int 2t dt) = e^{t^2}$ ,  $\frac{d}{dt}(e^{t^2}u) = 4te^{t^2} \Rightarrow e^{t^2}u(t) = 4 \int te^{t^2} dt + C$   
 $v = e^{t^2} \Rightarrow dv = 2te^{t^2} dt \Rightarrow 2 \int e^v dv$

$$\therefore u(t) = 2 + ce^{-t^2}$$

b.  $\frac{dy}{dx} + \frac{y}{3} = \frac{(1-2x)^{1/3}}{x}$ ,  $z = y^{-1/3} \Rightarrow y = z^{-3}$ ,  $\frac{dz}{dx} = -3y^{-4} \frac{dy}{dx}$ ,  $-3y^{-4} \frac{dy}{dx} - y^{-3} = 2x - 1 \Rightarrow \frac{dz}{dx} - z = 2x - 1$

$$\mu(x) = e^{-x}, \quad \frac{d}{dx}(e^{-x}z) = (2x-1)e^{-x} \Rightarrow e^{-x}z = \int (2x-1)e^{-x} dx = -(2x+1)e^{-x} + C$$

$$\therefore z(x) = y^{-1/3}(x) = -(2x+1) + ce^x, \quad y(x) = (ce^x - 2x - 1)^{-3/2}$$