Math 342A   Fall 2004

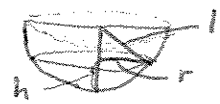
HW11 (11.1) 1, 2, 3, 4, 5, 6, 8, 9, 14, 15, 16

(4) $\frac{dT}{dt} = -k(T-20)$, $z = T - 20$ $\therefore$ $\frac{dz}{dt} = -kz$, $z(0) = T(0) - 20 = 180$ $\therefore$ $z(t) = T(t) - 20 = 180e^{-kt}$

$T(t) = 20 + 180e^{-kt}$, $T(6) = 100 = 20 + 180e^{-6k}$, $e^{6k} = \frac{180}{80}$ $\Rightarrow$ $k = \frac{1}{6}\ln(9/4) \approx 0.13516$

$T(t) = 25 = 20 + 180e^{-kt}$ $\Rightarrow$ $180e^{-kt} = 5$ or $e^{kt} = \frac{180}{5}$ $\therefore$ $t = \frac{1}{k}\ln(36) \approx 26.5\,\text{min}$ or 20.5 min later

(15) From 14, $\frac{dh}{dt} = -\frac{ac(2gh)^{1/2}}{A(h)}$, $a = 10^{-4}$, $c = 0.6$, $g = 9.81$. $A(h) = \pi r^2$, but



$r^2 = 1 - (1-h)^2 = 2h - h^2$, $\frac{dh}{dt} = -\frac{ac(2gh)^{1/2}}{\pi(2h-h^2)} = \frac{-6\times10^{-5}(19.62)^{1/2}h^{1/2}}{\pi(2h-h^2)}$

$\int(2h^{1/2} - h^{3/2})dh = -8.4596\times10^{-5}\int dt \Rightarrow \frac{4}{3}h^{3/2} - \frac{2}{5}h^{5/2} = -8.4596\times10^{-5}t + c \Leftrightarrow 20h^{3/2}(t) - 6h^{5/2}(t) = -0.001269t + C$

$h(0) = 1 \Rightarrow 20 - 6 = C$ $\therefore$ $20h^{3/2}(t) - 6h^{5/2}(t) = -0.001269t + 14$. $h(t) = 0 \Rightarrow 0.001269t = 14$ or

$t = 11033$ sec or 3.06 hr.

(11.2) 1, 2, 3, 4, 5, 6, 8, 10, 13, 15, 16, 20

(3) a. $y' + \frac{1}{x}y = 2$, $y(2) = 2$, $\mu(x) = \exp(\int \frac{1}{x}dx) = x$, $\frac{d}{dx}(xy) = 2x$, $xy(x) = x^2 + C \Rightarrow y(x) = x + \frac{C}{x}$, $y(2) = 2$

$\Rightarrow c = 0$ $\therefore$ $y(x) = x$

b. $y' + \tan(x)y = \cos^2 x$, $y(0) = -1$, $\mu(x) = \exp\{\int \frac{\sin(x)}{\cos(x)}dx\} = \frac{1}{\cos(x)}$, $\frac{d}{dx}\left(\frac{y}{\cos(x)}\right) = \cos(x)$, $\frac{y(x)}{\cos(x)} = \sin(x) + c$

$y(x) = \cos(x)\sin(x) + c\cos(x)$, $y(0) = -1 = c$ $\therefore$ $y(x) = \cos(x)\sin(x) - \cos(x)$

(15) a. $\frac{du}{dt} + 2tu = 4t$, $\mu(t) = \exp\int 2t\,dt = e^{t^2}$ $\therefore$ $\frac{d}{dt}(e^{t^2}u) = 4te^{t^2} \Rightarrow e^{t^2}u(t) = 4\int te^{t^2}dt + C$

$v = t^2$ $dv = 2t\,dt$

$2\int e^v dv$

$\therefore$ $u(t) = 2 + ce^{-t^2}$

b. $\frac{dy}{dx} + \frac{y}{3} = \frac{(1-2x)y^4}{3}$, $z = y^{1-4} = y^{-3}$, $\frac{dz}{dx} = -3y^{-4}\frac{dy}{dx}$, $-3y^{-4}\frac{dy}{dx} - y^{-3} = 2x - 1$ or $\frac{dz}{dx} - z = 2x - 1$

$\mu(x) = e^{-x}$, $\therefore$ $\frac{d}{dx}(e^{-x}z) = (2x-1)e^{-x} \Rightarrow e^{-x}z = \int(2x-1)e^{-x}dx = -(2x+1)e^{-x} + C$

$\therefore$ $z(x) = y^{-3}(x) = -(2x+1) + ce^{x}$, $y(x) = (ce^{x} - 2x - 1)^{-1/3}$

$19 + 20 = 39$