Math 342 A Fall 2004

HW 10 (10.3) 2, 3, 5, 7, 10, 15, 16, 17

(5) $\dot{x} = \begin{pmatrix} 2 & 5 \\ 1 & 6 \end{pmatrix} x$, $x(0) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. $\begin{vmatrix} 2-\lambda & 5 \\ 1 & 6-\lambda \end{vmatrix} = \lambda^2 - 8\lambda + 7 = (\lambda-1)(\lambda-7) = 0$. For $\lambda_1 = 1$,

$\begin{pmatrix} 1 & 5 \\ 1 & 5 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \xi_1 = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ For $\lambda_2 = 7$, $\begin{pmatrix} -5 & 5 \\ 1 & -1 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\xi_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\therefore \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = c_1 \begin{pmatrix} 5 \\ -1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{7t}$

$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} = c_1 \begin{pmatrix} 5 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\therefore$ $5c_1 + c_2 = 3$ & $-c_1 + c_2 = -2 \Rightarrow 6c_1 = 5$ $c_1 = 5/6 \Rightarrow c_2 = -\frac{7}{6}$

$\therefore \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 25/6 \\ -5/6 \end{pmatrix} e^{t} - \begin{pmatrix} 7/6 \\ 7/6 \end{pmatrix} e^{7t}$

(16) From 15: $\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} -1 & 1/4 & 0 \\ 1 & -3/2 & 1/2 \\ 0 & 1/2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, Let $\vec{x} = \vec{u} e^{i\omega t}$, $\ddot{\vec{x}} = -\omega^2 \vec{u} \cdot e^{i\omega t}$ $\begin{vmatrix} -1+\omega^2 & 1/4 & 0 \\ 1 & -\frac{3}{2}+\omega^2 & 1/2 \\ 0 & 1/2 & -1+\omega^2 \end{vmatrix} = 0$

$\therefore \omega^6 - \frac{7}{2}\omega^4 + \frac{7}{2}\omega^2 - 1 = (\omega^2 - 1)(\omega^2 - \frac{1}{2})(\omega^2 - 2) = 0$ $\therefore \omega^2 = \frac{1}{2}, 1, 2$. It follows that the fundamental frequencies are $\omega = \frac{1}{\sqrt{2}}, 1, \sqrt{2}$.

(10.5) 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 21

(5) $A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ For $\lambda_1 = 2$, $\begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\xi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. For $\lambda_{2,3} = 3$, $\begin{pmatrix} -1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\xi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\xi_3 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$ $S = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, Maple $S^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $S^{-1}AS = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(12) $A = \begin{pmatrix} -3 & 2 \\ -1 & 0 \end{pmatrix}$ $\begin{vmatrix} -3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = (\lambda+1)(\lambda+2) = 0$ For $\lambda_1 = -1$, $\xi_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

For $\lambda_2 = -2$, $\xi_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\therefore S = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ $S^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}$ $D = S^{-1}AS = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

$e^A = S e^D S^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-1} & 0 \\ 0 & e^{-2} \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -e^{-1} & 2e^{-1} \\ e^{-2} & -e^{-2} \end{pmatrix} = \begin{pmatrix} 2e^{-2} - e^{-1} & 2e^{-1} - 2e^{-2} \\ e^{-2} - e^{-1} & 2e^{-1} - e^{-2} \end{pmatrix}$

17 + 20 = 37