

HW 10 (10.3) 2, 3, 5, 7, 10, 15, 16, 17

$$(5) \dot{x} = \begin{pmatrix} 1 & 5 \\ 1 & 6 \end{pmatrix} x, x(0) = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \begin{vmatrix} 2-\lambda & 5 \\ 1 & 6-\lambda \end{vmatrix} = \lambda^2 - 7\lambda + 7 = (\lambda-1)(\lambda-7) = 0. \text{ For } \lambda_1 = 1,$$

$$\begin{pmatrix} 1 & 5 \\ 1 & 6 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \xi_1 = \begin{pmatrix} 5 \\ -1 \end{pmatrix} \text{ For } \lambda_2 = 7, \begin{pmatrix} -5 & 5 \\ 1 & -1 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \xi_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \therefore \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = c_1 \begin{pmatrix} 5 \\ -1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{7t}$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} = c_1 \begin{pmatrix} 5 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \therefore 5c_1 + c_2 = 3 \quad -c_1 + c_2 = -2 \Rightarrow \begin{cases} c_1 = 5 \\ c_2 = -7/6 \end{cases} \Rightarrow c_2 = -\frac{7}{6}$$

$$\therefore \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 25/6 \\ -5/6 \end{pmatrix} e^t - \begin{pmatrix} 7/6 \\ 7/6 \end{pmatrix} e^{7t}$$

$$(16) \text{ From 15: } \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -1 & 1/4 & 0 \\ 1 & -3/2 & 1/2 \\ 0 & 1/2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \text{ Let } \vec{x} = \vec{u} e^{i\omega t} \quad \begin{vmatrix} -1 + \omega^2 & 1/4 & 0 \\ 1 & -3/2 + \omega^2 & 1/2 \\ 0 & 1/2 & -1 + \omega^2 \end{vmatrix} = 0$$

$$\therefore \omega^6 - \frac{7}{2}\omega^4 + \frac{7}{2}\omega^2 - 1 = (\omega^2 - 1)(\omega^2 - \frac{1}{2})(\omega^2 - 2) = 0 \quad \therefore \omega^2 = \frac{1}{2}, 1, 2. \text{ It follows that the fundamental frequencies are } \omega = \frac{1}{\sqrt{2}}, 1, \sqrt{2}.$$

(10.5) 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 21

$$(5) A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{ For } \lambda_1 = 2, \begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \xi_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \text{ For } \lambda_{2,3} = 3, \begin{pmatrix} -1 & 0 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\xi_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \xi_3 = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \text{ Maple } S^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, S^{-1}AS = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$(12) A = \begin{pmatrix} -3 & 2 \\ -1 & 0 \end{pmatrix} \quad \begin{vmatrix} -3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = (\lambda+1)(\lambda+2) = 0 \text{ For } \lambda_1 = -1, \xi_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{For } \lambda_2 = -2, \xi_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \therefore S = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \quad S^{-1} = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} \quad D = S^{-1}AS = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$$

$$e^{At} = S e^{Dt} S^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -e^{-t} & 2e^{-t} \\ e^{-2t} & -e^{-2t} \end{pmatrix} = \begin{pmatrix} 2e^{-2t} - e^{-t} & 2e^{-t} - 2e^{-2t} \\ e^{-2t} - e^{-t} & 2e^{-t} - e^{-2t} \end{pmatrix}$$