October 1, 2004 Math 342A Name ____________________

1. Let $f(x, y, z) = z \arctan(y/x)$ and $\mathbf{V} = y \ln(x)\mathbf{i} + e^{yz}\mathbf{j} + z \cos(xz)\mathbf{k}$. Evaluate the following:

a. ∇f b. $\nabla \cdot \mathbf{V}$ c. $\nabla \times \mathbf{V}$ d. $\mathbf{V} \cdot \nabla \times \nabla f$

2. Given the position vector below, determine the velocity, speed, and acceleration, the tangential and normal components of the acceleration, the curvature, torsion, and the unit tangent, unit normal, and binormal vectors.

$$\mathbf{R}(t) = \frac{4}{9}(1+t)^{\frac{3}{2}}\mathbf{i} + \frac{4}{9}(1-t)^{\frac{3}{2}}\mathbf{j} + \frac{t}{3}\mathbf{k}$$

3. Suppose a particle is traveling along a path given by

$$\mathbf{r}(t) = \cosh(t)\mathbf{i} + \sinh(t)\mathbf{j} + t\mathbf{k}, \qquad t \in [0, 1].$$

a. Compute $\mathbf{v}(t) \cdot (\mathbf{r}(t) \times \mathbf{a}(t))$.
b. Find the total distance traveled by the particle.
c. If the particle is subjected to a force field with $\mathbf{F} = y \sin(z)\mathbf{i} + x \sin(z)\mathbf{j} + xy \cos(z)\mathbf{k}$, then find the total work done by $\mathbf{F}$ as the particle traverses its path.

4. Find the work done by

$$\mathbf{F} = \left(y \cos(x) - x^2(y+1)\right)\mathbf{i} + \left(\sin(x) + e^{y^2}\right)\mathbf{j}$$

for each of the following paths:
a. The path along the x-axis from 0 to 2.
b. The closed path starting along the path in part a., proceeding along $x^2 + y^2 = 4$ to the point (0,2), and returning along the y-axis to the origin.

5. Let $\mathbf{V} = y \ln(x)\mathbf{i} - (y^2/2x)\mathbf{j} + (z^2 + 1)\mathbf{k}$ be the flow of an incompressible fluid with density $\rho = 1$.
a. Consider the flow downward through the surface $S_1 = \{(x, y, z) : x^2 + y^2 \leq 4, z = 0\}$, and determine the mass of fluid flowing across S_1 by evaluating

$$\int\int_{S_1} \mathbf{V} \cdot \mathbf{n} dA.$$

b. Now determine the total mass of the fluid leaving the region bounded by $S = S_1 + S_2$, where $S_2 = \{(x, y, z) : x^2 + y^2 + z^2 \leq 4, z \geq 0\}$, by evaluating

$$\int\int_{S} \mathbf{V} \cdot \mathbf{n} dA.$$

c. Combine the results to determine how much fluid leaves through the top.

6. Consider the surface $S = \{(x, y, z) : z = x^2 + y^2, z \leq 4\}$.

a. Find the parametric representation of the surface $\mathbf{r}(u, v)$ and its surface normal vector $\mathbf{N}$.

b. Find the surface area of S,

$$\int\int_S dA.$$

c. Let C be the curve formed by the intersection of S with $z = 4$ (oriented clockwise when viewed by an observer at the origin), and let $\mathbf{F} = 2y\mathbf{i} + x\mathbf{j} - xze^y\mathbf{k}$. Evaluate

$$\oint_C \mathbf{F} \cdot d\mathbf{R}.$$