October 8, 2004 Math 342A Name ______________________

1. Let $f(x, y, z) = z\sin(x)\cosh(y)$ and $\mathbf{V} = e^{x^2y}\mathbf{i} + 2\ln|y|\mathbf{j} + y\cos(z)\mathbf{k}$. Evaluate the following:
a. ∇f b. $\nabla \cdot \mathbf{V}$ c. $\nabla \times \mathbf{V}$ d. $\nabla \cdot \nabla f = \nabla^2 f$.

2. Suppose a particle is traveling along a path given by

$$\mathbf{R}(t) = 2\cos(t/2)\mathbf{i} - 2\sin(t/2)\mathbf{j} + 2t\mathbf{k}, \qquad t \in [0, 3\pi].$$

a. Find the total distance traveled by the particle.

b. If the particle is subjected to a force field with $\mathbf{F} = [2x\cos(z) - y]\mathbf{i} + [3y^2 - x]\mathbf{j} - x^2\sin(z)\mathbf{k}$, then find the total work done by $\mathbf{F}$ as the particle traverses its path.

3. Find the work done by

$$\mathbf{F} = (xy^2 + ye^x)\mathbf{i} + (e^x - x^2y)\mathbf{j}$$

for each of the following paths:

a. The path along the y-axis from 3 to 0.

b. The straight line path from the origin to the point $(3/\sqrt{2}, 3/\sqrt{2})$.

c. The circular arc given by $\mathbf{R}(t) = 3\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j}$, $t \in [\pi/4, \pi/2]$. (Hint: You may want to take advantage of the fact that this third curve with the other two creates a closed path.)

4. Consider the elliptical surface Σ cut from the plane $z = cx$ by the cylinder $x^2 + y^2 = 1$.

a. Find the parametric representation of the surface $\mathbf{r}(u, v)$, including the limits on u and v. Give the normal to this surface.

b. Find the surface area of Σ,

$$\int\int_\Sigma d\sigma.$$

c. Let C be the curve formed by the edge of the surface Σ (oriented clockwise when viewed by an observer at the origin), and let $\mathbf{F} = x^2\sinh(z)\mathbf{i} + (x + 2ye^z)\mathbf{j} + y^2e^z\mathbf{k}$. Evaluate

$$\oint_C \mathbf{F} \cdot d\mathbf{R}.$$

5. Let $\mathbf{F} = y\mathbf{i} + xy\mathbf{j} + (z + x^2)\mathbf{k}$ be a vector field.

a. Find the downward flux of $\mathbf{F}$ across the surface $\Sigma_1 = \{(x, y, z) : x^2 + y^2 \le 4, z = 0\}$ by evaluating

$$\int\int_{\Sigma_1} \mathbf{F} \cdot \mathbf{N} d\sigma.$$

b. Find the net outward flux of $\mathbf{F}$ across the region bounded by the surface $\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_3$ with $\Sigma_2 = \{(x, y, z) : x^2 + y^2 = 4, 0 \le z \le 4\}$ and $\Sigma_3 = \{(x, y, z) : z = x^2 + y^2, 0 \le z \le 4\}$ by evaluating

$$\int\int_\Sigma \mathbf{F} \cdot \mathbf{N} d\sigma.$$