

1. a. The population of Canada was 24,070,000 in 1980, while in 1990 it was 26,620,000. Assuming the population is growing according to the principle of Malthusian growth (with no food or space limitations), find the population as a function of time, and determine its doubling time.
b. For the same years, the populations of Kenya were 16,681,000 and 24,229,000, respectively. Find the population as a function of time.
c. In what year do the populations of Canada and Kenya become equal?
2. When Strontium-90 (^{90}Sr) is ingested, it can displace calcium in the formation of bones. After a β decay it becomes an isotope of krypton (an inert gas), and diffuses out of the bone, leaving the bones porous.
 - a. Suppose that a particular bone contains 20 grams of ^{90}Sr , which has a half-life of 28 years. Write an equation describing the amount of ^{90}Sr remaining at any time, and determine the amount after 10 years.
 - b. Find how long until only 7 grams of ^{90}Sr remain.
3. You have just boiled a new batch of broth for your important culture of *E. coli*, so it is at 100°C . You have it sitting in a room that is at 22°C , and you find 5 minutes later that it's cooled to 93°C . You want to inoculate the culture when it reaches 40°C . You are interested in knowing if you can safely go off to exercise while the broth cools.
 - a. Let $T(t)$ be the temperature of the broth. Assume that the broth satisfies Newton's Law of Cooling, and set up the differential equation for the temperature of the broth, and solve it.
 - b. Find how long it will be until you need to inoculate the broth with your culture. Sketch a graph of the function of $T(t)$ for the first hour showing its starting and ending temperatures for the hour.
4. a. *Paramecia* in a pond sample are growing according to the principle of Malthusian growth (with no food or space limitations). Initially, there are 1500 *Paramecia*. Four hours later, the population has 2000 individuals. Find the population of *Paramecia* as a function of time, and determine its cell doubling time.
b. A large population of 5000 is transferred to where a limited diet affects their growth dynamics. The *Paramecia* now satisfy the population dynamics of the differential equation:

$$\frac{dP}{dt} = -0.1P + 100.$$

Solve this differential equation and find what happens to the population as $t \rightarrow \infty$.

5. A well mixed pond, $V = 200,000 \text{ m}^3$, is initially clean ($c(0) = 0$). A polluted stream with a concentration of dioxin, $Q = 5 \text{ ppb}$ enters the pond, flowing at a rate of $f = 4000 \text{ m}^3/\text{day}$. Another stream carries the water away at the same rate.
 - a. Find a differential equation for this problem and solve it.
 - b. Find how long before the pond has a concentration of 4 ppb.
 - c. Find the limiting concentration.