1. The population of China in 1980 was about 985 million, and a census in 1990 showed that the population had grown to 1,137 million. Assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1+r)P_n,$$

where n is the number of decades after 1980 and P_n is population n decades after 1980.

a. Use the data above to find the growth constant r and then write the general solution P_n. Predict the population in the year 2000.

b. How long does it take for China's population to double?

2. Take $r = 0.15$ and $P_0 = 75,994,575$ (the population in 1900). Use the Malthusian growth model

$$P_{n+1} = (1+r)P_n,$$

where n is the number of decades after 1900 and P_n is population n decades after 1900. Simulate this model for $n = 1, 2, 3, ..8, 9$ to estimate the population through the 20^{th} century. Compare your results to the actual census data by computing the error at each decade. Also, determine how long the model predicts for the population to double and compare this to the actual data.

3. An invertebrate living in a pond is effected by a pollutant that is slowly seeping into the ecosystem. The population dynamics for this invertebrate is given by the nonautonomous Malthusian growth model

$$P_{n+1} = (1 + k(t_n))P_n \quad \text{with} \quad P_0 = 40,000,$$

where $t_n = n$ is the number of days from the initial measurement of the population and $k(t) = 0.08 - 0.01t$ is the growth rate of this invertebrate, which is clearly declining as t increases.

a. Find the population for this organism for the first 5 days.

b. When the growth rate falls to zero, this population reaches its maximum. Find when this occurs and what the population is at that time.

c. Determine when the pollution level gets so high that this invertebrate goes extinct.

4. In the model for breathing, we could also have kept track of the Nitrogen (N_2) in the exhaled breath also. The mathematical model is the same as in the lecture notes,

$$c_{n+1} = (1-q)c_n + q\gamma.$$

For the normal subject, we found that $q = 0.18$. The percent of N_2 in the atmosphere is 78%, so this gives $\gamma = 0.78$. Assume that the initial concentration of N_2 in the lungs is given by $c_0 = 0.7$. Find c_1, c_2, and c_3. Also, find the equilibrium point, c_e. Does the solution approach the equilibrium (stable) or move away from the equilibrium (unstable)?

5. The population in the U. S. at the turn of the last century is given in the following table (with population in millions).

Year	1900	1910	1920	1930
Population	76.0	92.0	105.7	122.8

a. Let $p_0 = 76.0$ and consider the Malthusian growth model

$$p_{n+1} = 1.17p_n,$$

where n is in decades. Find p_1, p_2, and p_3. Determine the percent error in these predictions compared to the actual values.

b. Again let $p_0 = 76.0$ and consider the Malthusian growth model with immigration. Assume that the immigration over a decade is approximately 3.0 million, then the model is given by

$$p_{n+1} = 1.14p_n + 3.0,$$

where n is in decades. Find p_1, p_2, and p_3. Determine the percent error in these predictions compared to the actual values. Notice that the actual growth rate is 3% lower in this model.

6. Below is data on several populations of herbivores in related areas.

p_0	p_1
70	90
100	150
150	250

The data is assumed to fit a discrete Malthusian model with emigration in the form

$$p_{n+1} = rp_n - \mu,$$

where $r - 1$ is the growth rate and μ is the emigration rate.

a. Use the data below to determine the updating function for this population, i.e., find r and μ and write the equation for this model.

b. Beginning with $p_0 = 100$, find the populations p_1, p_2, and p_3.

c. Find the equilibrium value and determine the stability of this equilibrium.

7. Consider the discrete logistic growth model given by

$$P_{n+1} = f(P_n) = 1.25P_n - 0.00125P_n^2.$$

a. Suppose that the initial population $P_0 = 2000$. Find the population of the next three generations, P_1, P_2, and P_3. Find all equilibria and determine their stability.

b. Sketch a graph of the updating function, $f(P)$, with the identity map, $P_{n+1} = P_n$. Find the intercepts and the vertex of the parabola.

8. A modified version of the discrete logistic growth model that includes emigration is given by

$$P_{n+1} = f(P_n) = 1.1P_n - 0.0001P_n^2 - 9.$$

a. Suppose that the initial population P_0 is 500. Find the population of the next three generations, P_1, P_2, and P_3.

b. Sketch a graph of the updating function with the identity map, $P_{n+1} = P_n$. Be sure to show the intercepts of the parabola as well as the vertex. Find the equilibria and identify them on your graph. Determine the stability of the equilibria.

9. Consider Hassell's model that is given by

$$p_{n+1} = H(p_n) = \frac{10p_n}{1 + 0.0001p_n^2}.$$

a. Assume that $p_0 = 100$ and find the population for the next three generations, p_1, p_2, and p_3.

b. Find the p-intercepts and the horizontal asymptote for $H(p)$ and sketch a graph of $H(p)$ for $p > 0$ along with the identity map, $p_{n+1} = p_n$.

c. By solving $p_e = H(p_e)$, determine all equilibria for this model and determine their stability.

10. Many biologists in fishery management use Ricker's model to study the population of fish. Let P_n be the population of fish in any year n, then Ricker's model is given by

$$P_{n+1} = R(P_n) = aP_n e^{-bP_n}.$$

Suppose that the best fit to a set of data gives $a = 8$ and $b = 0.002$ for the number of fish sampled from a particular river.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of $R(P)$ with the identity function, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

11. In fishery management, it is important to know how much fishing can be done without severely harming the population of fish. A modification of Ricker's model that includes fishing is given by the model:

$$P_{n+1} = F(P_n) = aP_n e^{-bP_n} - hP_n,$$

where $a = 4$ and $b = 0.002$ are the constants in Ricker's equation that govern the dynamics of the fish population without any fishing and h is the intensity of harvesting fish.

a. Let $h = 0.5$ and $P_0 = 100$, then find P_1, P_2, and P_3.

b. With $h = 0.5$, find all equilibria for this model and describe the behavior of these equilibria.

c. Find all equilibria for this model and describe the behavior of these equilibria when $h = 1$ and $h = 2$.

d. How intense can the fishing be before this population of fish is driven to extinction? That is, find the value of h that makes the only equilibrium be zero (or less than zero).

12. Consider the chalone model for mitosis given by the equation

$$P_{n+1} = f(P_n) = \frac{2P_n}{1 + (bP_n)^c},$$

where $b = 0.05$ and $c = 2$.

a. Let $P_0 = 10$, then find P_1, P_2, and P_3.

b. Sketch a graph of $f(P)$ with the identity function for $P \geq 0$, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

13. Biology 354 (Ecology and Evolution) uses the discrete logistic growth model

$$P_{n+1} = f(P_n) = P_n + rP_n\left(1 - \frac{P_n}{M}\right).$$

This problem explores some of the complications that can arise as the parameter r varies.

a. Let $M = 5,000$. Find all equilibria and determine the stability as a function of r.

b. Let $r = 1.89$ with $P_0 = 2,000$. Simulate the discrete dynamical system for 50 generations. Make a table listing the population for every fifth generation (P_0, P_5, ... P_{10}). Graph the solution of the dynamical system and write a brief description of what you observe in your solution.

c. Repeat the process in Part b. with $r = 2.1$ and $r = 2.62$. (You can make a separate table for these simulations or simply add these to your table in Part b. with appropriate labeling.) Don't forget to write a description of these solutions and how they compare to each other and your solution in Part b. What behavior do you observe for the solution in relation to the larger of the two equilibria?

d. Find a parameter value that gives you an oscillation with period 3. (You may want to use the applet in the notes to find this value.) This always implies that the dynamical system has gone through chaos.