1. a. $y(t) = Ae^{t^3}$,

b. $y(t) = \pm\sqrt{t^2 + C}$,

c. $y(t) = Ce^{-2t} + \dfrac{3}{2}$,

d. $y(t) = \ln\left(\dfrac{t^2}{2} + 2t + C\right)$,

e. $y(t) = -\dfrac{1}{t^3 + C}$,

f. $y(t) = \pm\sqrt{t - t^2 + C}$,

g. $y(t) = \pm\sqrt{\dfrac{t^2}{2} + 4\ln|t| + C}$,

h. $y(t) = t^3 + 12t + C$.

2. a. $y(t) = 5e^{t^2}$,

b. $y(t) = \sqrt{t^2 + t + 1}$,

c. $y(t) = \dfrac{-1 + \sqrt{4t^2 - 15}}{2}$,

d. $y(t) = 5 - 3e^{-t}$,

e. $y(t) = 3e^{-2t}$,

f. $y(t) = 4t^2$.

g. $y(t) = \dfrac{1}{1 - \sin(2t)}$,

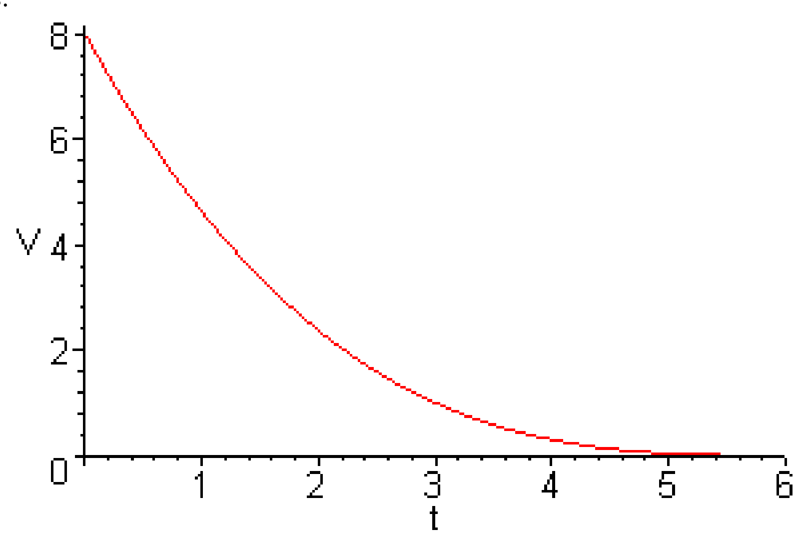
h. $y(t) = 5e^{1-\cos(t)}$.

3. a. The solution is $V(t) = \left(2 - \frac{k}{3}t\right)^3$ with $k = 1$ or

$$V(t) = \left(2 - \frac{t}{3}\right)^3.$$

b. The mothball lasts 6 months.

c. The graph of $V(t)$ is below.



4. a. The volume of the cell is given by $V(t) = (0.01t + 1)^4$.

b. The cell to doubles its volume in $100\left(2^{1/4} - 1\right) \simeq 18.92$ time units.

5. a. The general solution for the growing raindrop is given by

$$V(t) = \left(\frac{kt + C}{3}\right)^3.$$

b. The specific solution becomes $V(t) = \left(\frac{0.1\,t+3}{3}\right)^3$. This solution grows to 8 units in $t = 30$ time units.

6. a. This is a linear differential equation with the solution $L(t) = 72 - 71e^{-0.09t}$. The limit for large t is

$$\lim_{t\to\infty} L(t) = 72.$$

b. The walleye reaches 45 cm at $t = \frac{100}{9}\ln\left(\frac{71}{27}\right) \simeq 10.74$ (yr).

7. a. The expression for $C(t)$ as a function of $N(t)$ is

$$C(t) = 2\sqrt{\frac{\pi N(t)}{k}}.$$

b. The assumption is that the rate of spread of the disease (which is the change in number of diseased trees) is proportional to the circumference of the infected region. Since $C(t)$ is the circumference (and we chose q to be the proportionality factor), the differential equation is given by

$$\frac{dN}{dt} = qC, \qquad N(0) = 1,$$

assuming we start with one infected tree in the middle of the grove.

c. The solution for this model for the spread of a disease in an orchard is given by

$$N(t) = \left(q\sqrt{\frac{\pi}{k}}t + 1\right)^2.$$

8. a. The solution is given by

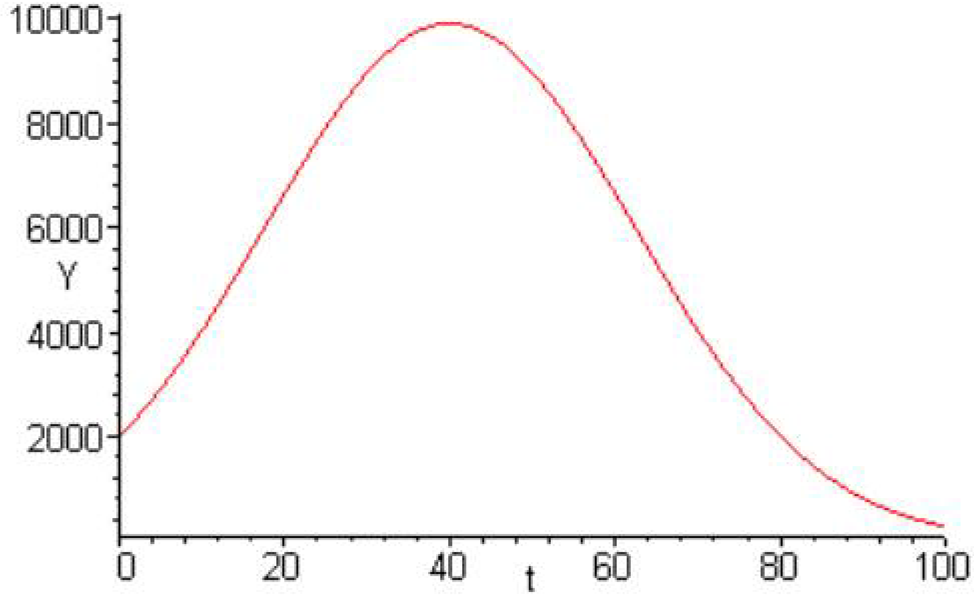
$$Y(t) = 2000e^{0.08t}.$$

It takes $t = \frac{25}{2}\ln(2) \simeq 8.664$ hrs for the population to double.

b. The solution is given by

$$Y(t) = 2000e^{0.08t-0.001t^2}.$$

c. The maximum population occurs at $t = 40$ hrs with a maximum population of $Y(40) = 9906$. The population returns to 2000 when $t = 80$ hrs. A graph of the solution is seen below



9. a. The solution to the differential equation is

$$P(t) = 6.94\, e^{bt-\frac{1}{2}at^2},$$

where $a = 0.0001017$ and $b = 0.004697$.

b. The model gives $P(50) = 7.72$ million in 2000, which is a -4.9% error from the actual census data.

c. The model predicts that the maximum population for Austria is 7.735 million when $t = 46.2$ or sometime in 1996. Clearly this is off because of the 2000 census data.

10. a. For convenience, let 1941 correspond to $t = 0$ and define $M(t)$ to be the population of India. The Malthusian growth model is

$$M(t) = 319e^{kt},$$

where $k = \frac{1}{20}\ln\left(\frac{439}{319}\right) \simeq 0.015965$. The population from this model for 1951 is $M(10) = 374.2$ million. The percent error is 3.7%.

b. The nonautonomous Malthusian growth model is

$$P(t) = 319e^{0.00035967t^2+0.0087720t}.$$

c. The Malthusian growth model gives the population in 1991 as $M(50) = 708.7$ million, while the nonautonomous Malthusian growth model gives $P(50) = 1,215.6$ million. The percent error from the actual population of 846 million for the Malthusian growth model is 16.2%, while the percent error for the nonautonomous Malthusian growth model is 43.7%. So in this case, the Malthusian growth model is the better model. Below is a graph of both models and the data.