1. Solve the following differential equations. Note that you may want to use techniques from the previous sections to solve these equations.

a. $\dfrac{dy}{dt} = 3t^2 y,$

b. $\dfrac{dy}{dt} = \dfrac{t}{y},$

c. $\dfrac{dy}{dt} = 3 - 2y,$

d. $\dfrac{dy}{dt} = (t+2)e^{-y},$

e. $\dfrac{dy}{dt} = 3t^2 y^2,$

f. $\dfrac{dy}{dt} = \dfrac{(1-2t)}{2y},$

g. $2ty\dfrac{dy}{dt} = t^2 + 4,$

h. $\dfrac{dy}{dt} = 3t^2 + 12.$

2. Solve the following initial value problems.

a. $\dfrac{dy}{dt} = 2ty, \quad y(0) = 5,$

b. $2y\dfrac{dy}{dt} = 2t + 1, \quad y(0) = 1,$

c. $(1+2y)\dfrac{dy}{dt} = 2t, \quad y(2) = 0,$

d. $\dfrac{dy}{dt} = 5 - y, \quad y(0) = 2,$

e. $\dfrac{dy}{dt} = -2y, \quad y(0) = 3,$

f. $t\dfrac{dy}{dt} = 2y, \quad y(1) = 4.$

g. $\dfrac{dy}{dt} = 2\cos(2t)\, y^2, \quad y(0) = 1,$

h. $\dfrac{dy}{dt} = y\,\sin(t), \quad y(0) = 5,$

3. A mothball with volume $V(t)$ has an initial volume $V(0) = 8$ cm^3. The mothball slowly evaporates according to the differential equation

$$\frac{dV}{dt} = -kV^{2/3}.$$

After three months, the volume of the mothball has decreased to 1 cm^3, *i.e.*, $V(3) = 1$.

a. Solve this differential equation and find k.

b. Find how long it takes for the mothball to disappear.

c. Sketch a graph of $V(t)$.

4. Suppose that an empirical study of the growth of a yeast cell indicates that the volume of the cell satisfies the differential equation

$$\frac{dV}{dt} = 0.04V^{3/4}, \qquad V(0) = 1.$$

a. Solve this initial value problem.

b. Find how long it takes for the cell to double its volume.

5. It has been observed that the volume of a raindrop increases at a rate proportional to its surface area, *i.e.*, the change in volume, V, is proportional to $V^{2/3}$.

a. Set up a differential equation describing the growth of a raindrop and solve it.

b. If $V(0) = 1$ and the proportionality constant $k = 0.1$, find the time required for the volume to increase to 8.

6. It has been found that walleye growing in Northern lakes satisfy the von Bertalanffy equation for the growth of fish. Let L(t) be the length of a walleye in cm, then the equation for their length is given by

$$\frac{dL}{dt} = 0.09(72 - L), \qquad L(0) = 1.$$

a. Solve this differential equation and determine the maximum length of a walleye.

b. If a walleye must be 45 cm to reach sexual maturity, then determine how long it takes for these fish to mature.

7. Consider the following model for the spread of disease in an orchard. Assume that the disease spreads radially at a rate proportional to the circumference of the circular region of the diseased trees.

a. Assume that the number of infected trees in any given area is proportional to the area, so if $N(t)$ represents the number of infected trees and $r(t)$ is radius of the circular infected region, then $N(t) = k\pi x^2(t)$. The circumference of the infected region is given by $C(t) = 2\pi x(t)$. Find an expression for C as a function of N by eliminating $x(t)$ from the expressions for $C(t)$ and $N(t)$.

b. Explain why a reasonable model for the spread of the infection is given by the differential equation

$$\frac{dN}{dt} = qC, \qquad N(0) = 1.$$

c. From the information above, we have the model for the spread of a disease in an orchard is given by

$$\frac{dN}{dt} = 2q\sqrt{\frac{\pi}{k}}N^{1/2}, \qquad N(0) = 1.$$

Solve this differential equation.

8. a. A population of yeast is growing according to a Malthusian growth model. Suppose that it satisfies the initial value problem

$$\frac{dY}{dt} = 0.08\,Y, \qquad Y(0) = 2000,$$

where t is in hours. Solve this differential equation and determine how long it takes for this population to double.

b. Because of competition from another organism in the broth, the yeast has dwindling supplies of food for growth. An approximate model with a time varying growth rate from this competition is given by the following:

$$\frac{dY}{dt} = (0.08 - 0.002\,t)\,Y, \qquad Y(0) = 2000.$$

Solve this differential equation.

c. Find the maximum of this population and when this occurs. Also, determine when the population returns to 2000. Sketch a graph for this population.

9. Most of the Western European countries are having a dramatic decline in their growth rate to the point where their populations will actually begin to decline early in this century. Consider the case of Austria. Its population was 6.94 million in 1950, 7.47 million in 1970, and 7.72 million in 1990.

a. Use the nonautonomous Malthusian growth model given by

$$\frac{dP}{dt} = (b - at)P.$$

Let t be the number of years after 1950, then solve this differential equation. Use the population data for Austria to find the constants a and b.

b. The population for Austria was 8.13 million in 2000. Use the model above to estimate the population of Austria, then compute the percent error from the actual census data.

c. When does the model predict that Austria will have its largest population and what is that population?

10. For many years, the population of India has accelerated in its growth to where soon India will be the world's most populous country. The population of India in 1941 was 319 million, in 1951 it was 361 million, and in 1961 it was 439 million.

a. Use the data in 1941 and 1961 to find a Malthusian growth model for India's population. Find the percent error between this model and the actual data in 1951.

b. Consider the nonautonomous Malthusian growth model given by the differential equation

$$\frac{dP}{dt} = (at + b)P,$$

where the constants a and b are to be determined by the data from the years 1941, 1951, and 1961. Solve this differential equation with the data above.

c. If the population of India was 846 million in 1991, then use each of these models to estimate the population in 1991 and determine the error between the models and the actual census values. Which model provides the better estimate? Graph the solutions of the two models and the data points from 1941 to 1991.