1. a. With $n = 2$ and $x \in [0, 2]$, the midpoints of the subintervals are $x_1 = \frac{1}{2}$ and $x_2 = \frac{3}{2}$ with $\Delta x = 1$, so the midpoint rule gives

$$\int_0^2 (4 + 2x^2)dx \simeq \left(\left(4 + 2\left(\frac{1}{2}\right)^2\right) + \left(4 + 2\left(\frac{3}{2}\right)^2\right)\right) \cdot 1 = 13.$$

With $n = 2$, the trapezoid rule gives

$$\int_0^2 (4 + 2x^2)dx \simeq \left(\frac{1}{2}\left(4 + 2(0)^2\right) + (4 + 2(1)^2) + \frac{1}{2}(4 + 2(2)^2)\right) \cdot 1 = 14.$$

b. With $n = 4$, the subintervals have length $\Delta x = \frac{1}{2}$, so the midpoint rule gives

$$\begin{aligned}\int_0^2 (4 + 2x^2)dx &\simeq \left(\left(4 + 2\left(\frac{1}{4}\right)^2\right) + \left(4 + 2\left(\frac{3}{4}\right)^2\right) + \left(4 + 2\left(\frac{5}{4}\right)^2\right) + \left(4 + 2\left(\frac{7}{4}\right)^2\right)\right) \cdot \frac{1}{2} \\ &= 13.25.\end{aligned}$$

With $n = 4$, the trapezoid rule gives

$$\begin{aligned}\int_0^2 (4 + 2x^2)dx &\simeq \left(\frac{1}{2}\left(4 + 2(0)^2\right) + \left(4 + 2\left(\frac{1}{2}\right)^2\right) + (4 + 2(1)^2) + \left(4 + 2\left(\frac{3}{2}\right)^2\right) + \frac{1}{2}(4 + 2(2)^2)\right) \cdot \frac{1}{2} \\ &= 13.5.\end{aligned}$$

c. For $n = 2$, the midpoint rule has a $100\left(\frac{13-40/3}{40/3}\right) = -2.5\%$ error, which is a low estimate. The trapezoid rule has a $100\left(\frac{14-40/3}{40/3}\right) = 5.0\%$ error, which is a high estimate. Similarly, for $n = 4$, the midpoint rule has a -0.625% error, which is a low estimate. The trapezoid rule has a 1.25% error, which is a high estimate.

4. a. Since $f(0) = 8$, the y-intercept is $(0, 8)$. From $8 + 2x - x^2 = -(x + 2)(x - 4) = 0$, the x-intercepts are $(-2, 0)$ and $(4, 0)$. The midpoint between the x-intercepts is $x = 1$ with $f(1) = 9$, so the vertex is $(1, 9)$. See the graph on the other solution page.

b. With $n = 4$ and $x \in [0, 4]$, the midpoints of the subintervals are $x_1 = \frac{1}{2}$, $x_2 = \frac{3}{2}$, $x_3 = \frac{5}{2}$, and $x_4 = \frac{7}{2}$ with $\Delta x = 1$, so the midpoint rule gives

$$\begin{aligned}\int_0^4 (8 + 2x - x^2)dx &\simeq \left(\left(8 + 2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2\right) + \left(8 + 2\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2\right) + \left(8 + 2\left(\frac{5}{2}\right) - \left(\frac{5}{2}\right)^2\right)\right. \\ &\quad \left. + \left(8 + 2\left(\frac{7}{2}\right) - \left(\frac{7}{2}\right)^2\right)\right) \cdot 1 = 27.0.\end{aligned}$$

With $n = 4$, the trapezoid rule gives

$$\begin{aligned}\int_0^4 (8+2x-x^2)dx &\simeq \left(\frac{1}{2}\left(8+2(0)-(0)^2\right)+\left(8+2(1)-(1)^2\right)+\left(8+2(2)-(2)^2\right)\right.\\ &\left.+\left(8+2(3)-(3)^2\right)+\frac{1}{2}\left(8+2(4)-(4)^2\right)\right)\cdot 1 = 26.0.\end{aligned}$$

c. Simpson's rule gives the exact value with

$$\begin{aligned}\int_0^4 (8+2x-x^2)dx &= \left(\left(8+2(0)-(0)^2\right)+4\left(8+2(1)-(1)^2\right)+2\left(8+2(2)-(2)^2\right)\right.\\ &\left.+4\left(8+2(3)-(3)^2\right)+\left(8+2(4)-(4)^2\right)\right)\cdot\frac{1}{3} = 26.6667.\end{aligned}$$

For $n = 4$, the midpoint rule has a 1.25% error, which is a high estimate. The trapezoid rule has a -2.5% error, which is a low estimate.