1. a. With $n = 2$, the midpoint rule gives

$$\int_0^2 (4 + 2x^2)dx \simeq 13.$$

With $n = 2$, the trapezoid rule gives

$$\int_0^2 (4 + 2x^2)dx \simeq 14.$$

b. With $n = 4$, the midpoint rule gives

$$\int_0^2 (4 + 2x^2)dx \simeq 13.25.$$

With $n = 4$, the trapezoid rule gives

$$\int_0^2 (4 + 2x^2)dx \simeq 13.5.$$

c. For $n = 2$, the midpoint rule has a -2.5% error, which is a low estimate. The trapezoid rule has a 5.0% error, which is a high estimate. For $n = 4$, the midpoint rule has a -0.625% error, which is a low estimate. The trapezoid rule has a 1.25% error, which is a high estimate.

2. a. With $n = 4$, the midpoint rule gives

$$\int_0^2 x^4 dx \simeq 6.0703.$$

With $n = 4$, the trapezoid rule gives

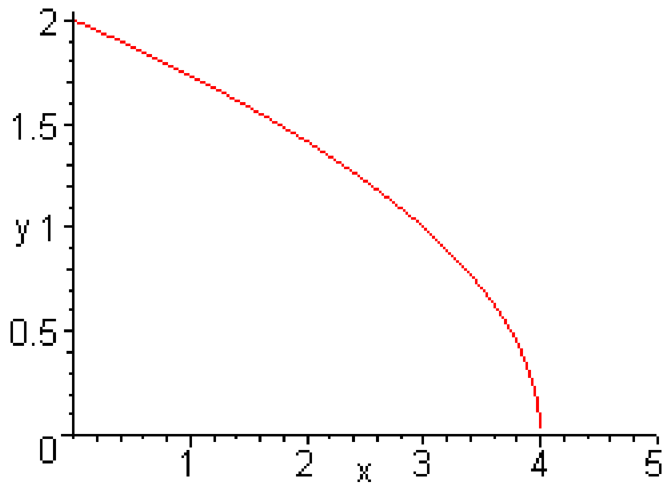
$$\int_0^2 x^4 dx \simeq 7.0625.$$

With $n = 4$, the Simpson's rule gives

$$\int_0^2 x^4 dx \simeq 6.4167.$$

b. The midpoint rule has a -5.15% error, which is a low estimate. The trapezoid rule has a 10.35% error, which is a high estimate. Simpson's rule has a 0.26% error, which is a high estimate.

3. a. The domain is $x \leq 4$. The graph is below.



b. With $n = 4$, the midpoint rule gives

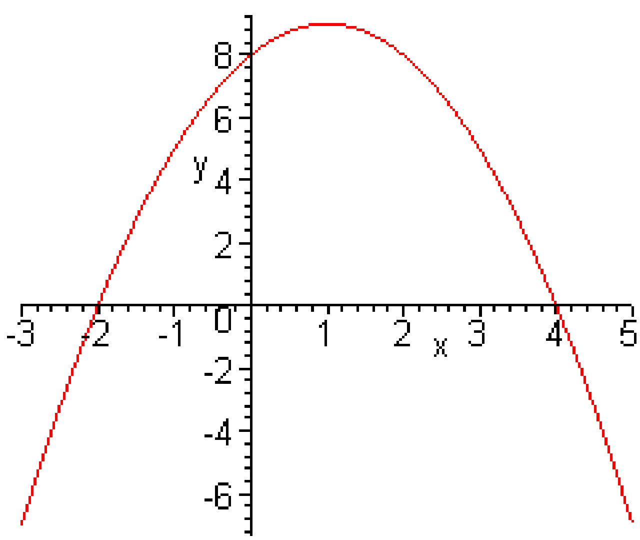
$$\int_0^4 \sqrt{4-x}dx \simeq 5.3838.$$

With $n = 4$, the trapezoid rule gives

$$\int_0^4 \sqrt{4-x}dx \simeq 5.1463.$$

The actual value is 5.3333.

4. a. The x-intercepts are $(-2, 0)$ and $(4, 0)$, and the y-intercept is $(0, 8)$. The vertex is $(1, 9)$. The graph is below.



b. With $n = 4$, the midpoint rule gives

$$\int_0^4 (8 + 2x - x^2)dx \simeq 27.0.$$

With $n = 4$, the trapezoid rule gives

$$\int_0^4 (8 + 2x - x^2)dx \simeq 26.0.$$

c. Simpson's rule gives the exact value with

$$\int_0^4 (8 + 2x - x^2)dx = 26.6667.$$

For $n = 4$, the midpoint rule has a 1.25% error, which is a high estimate. The trapezoid rule has a -2.5% error, which is a low estimate.

5. a. The average population is 20.875.

b. With the trapezoid rule, we obtain

$$P_{ave} = \frac{1}{7} \int_0^7 P(t)dt \simeq 21.5.$$

This answer is slightly higher as the endpoints aren't weighted as highly.

6. The cumulative dose from the trapezoid rule is

$$\int_0^{10} A(t)\, dt \simeq 0.70.$$

7. The area of the rectangle $f(x_i)\Delta x$, while the area of the triangle is $\frac{1}{2}(f(x_{i+1}) - f(x_i))\Delta x$. Combining these formulae gives the area of the trapezoid as $\frac{1}{2}(f(x_i) + f(x_{i+1}))\Delta x$. The trapezoid rule reflects this formula in that each point of the x-grid, except the endpoints, has its height measured twice (trapezoid to the right and the left). So the formula simply adds these trapezoid areas together.